# Public Key Encryption with Equality Test from Tag-Based Encryption

OMasayuki Tezuka

Keisuke Tanaka

Institute of Science Tokyo

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# Background of PKEET

#### Public Key Encryption with Equality Test (PKEET) [YTHW10]

$$ct_1 \leftarrow Enc(ek_1, pt_1)$$
 
$$ct_2 \leftarrow Enc(ek_2, pt_2)$$
 Tester

This test can be applied ciphertexts generated by different encryption keys.

#### Problem in Original PKEET by Yang et al. [YTHW10]



Anyone can be a tester and perform equality tests.

Problem

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For example, there is ct on an unknown plaintext pt.

Anyone can freely choose pt' and generates its ciphertext ct'.

#### Restricting Equality Tests with Trapdoor Delegation

Trapdoor was introduced to limit who can perform equality tests.



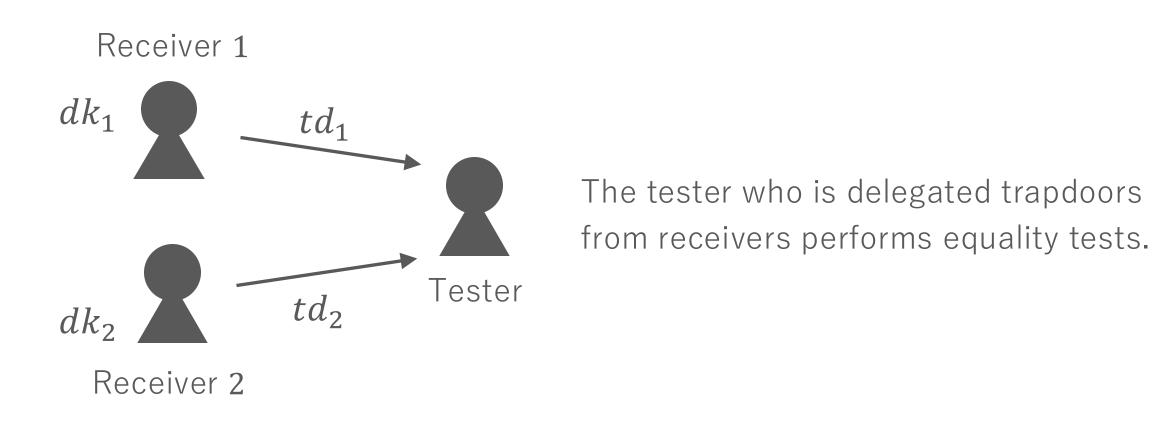






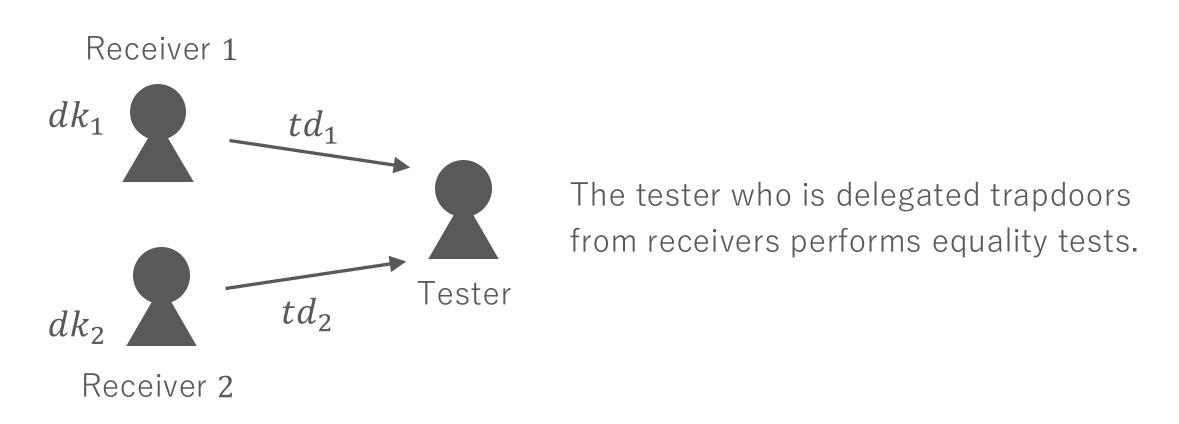
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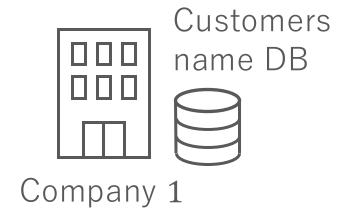


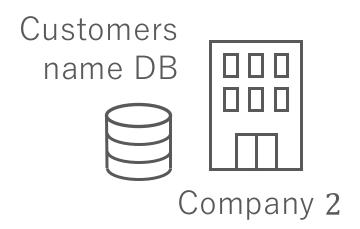
From now on, we consider this type of PKEET.

## Application of PKEET

- Keyword search on encrypted data
- Encrypted data partitioning
- Personal health record system
- Encrypted Database

#### Application of PKEET: Encrypted Database

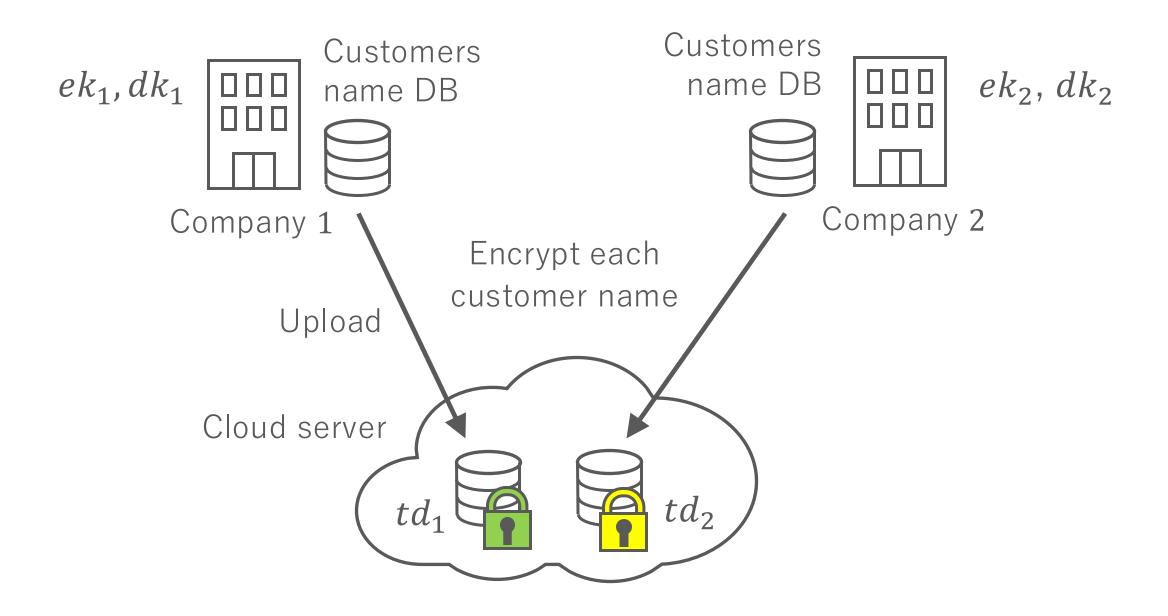




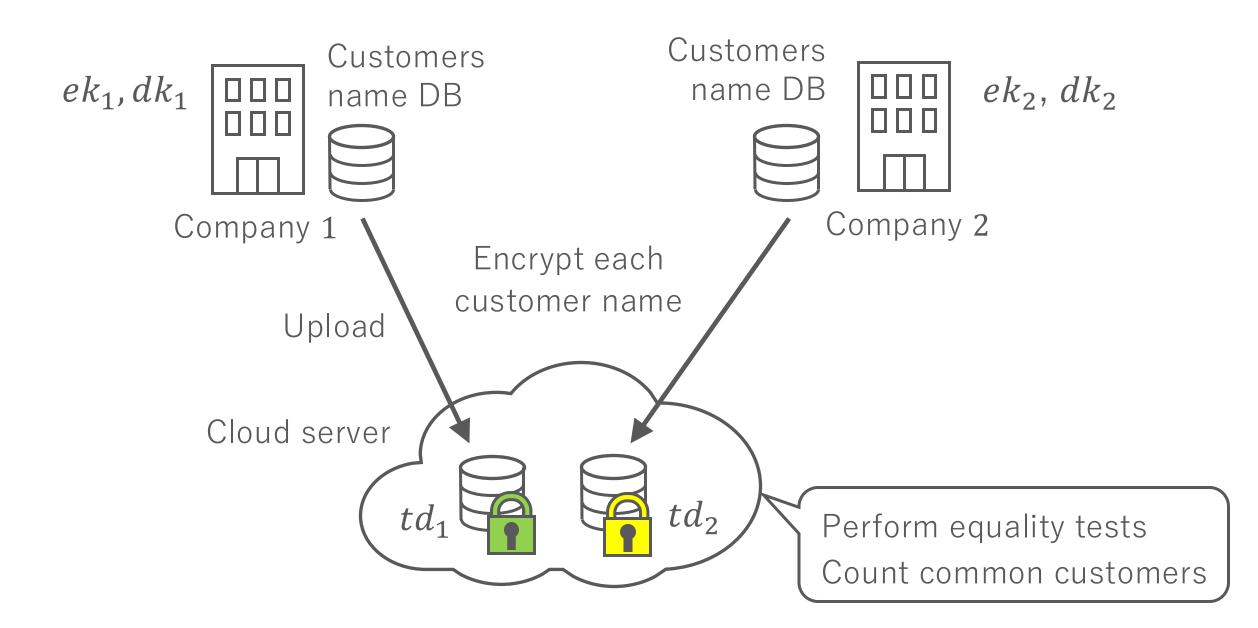
A data analysist want to know the number of common customers between two companies while maintaining privacy.

PKEET is useful for this scenario.

### Application of PKEET: Encrypted Database



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#### PKEET Constructions from Various Assumption

Pairing Based Constructions

e.g. [YTHW10], [Tan11]

Lattice-Based Constructions

e.g. [DFKRS19], [DRSFKS22]

We focus on generic constructions.

Generic Constructions

e.g. [LSQ18]. [LLSW19]

Scheme	Primitives	Without ROM
[LSQ18]	IND-CCA PKE	NO
[LLSW19]	IND-CCA PKE	NO
[LLSWY20]	sID-CPA HIBE + OTS + OW&CR Hash	Yes
[CPL25]	OW-CPA PKE	NO

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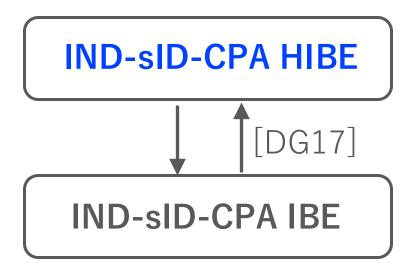
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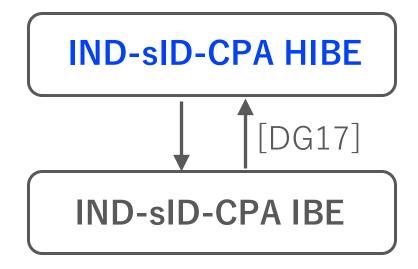
Question

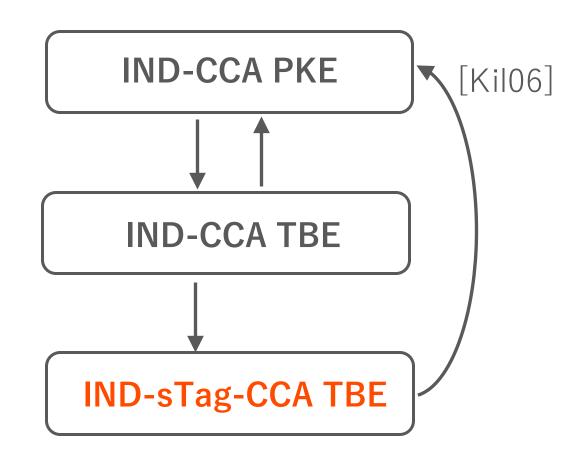
Can we give a PKEET scheme from

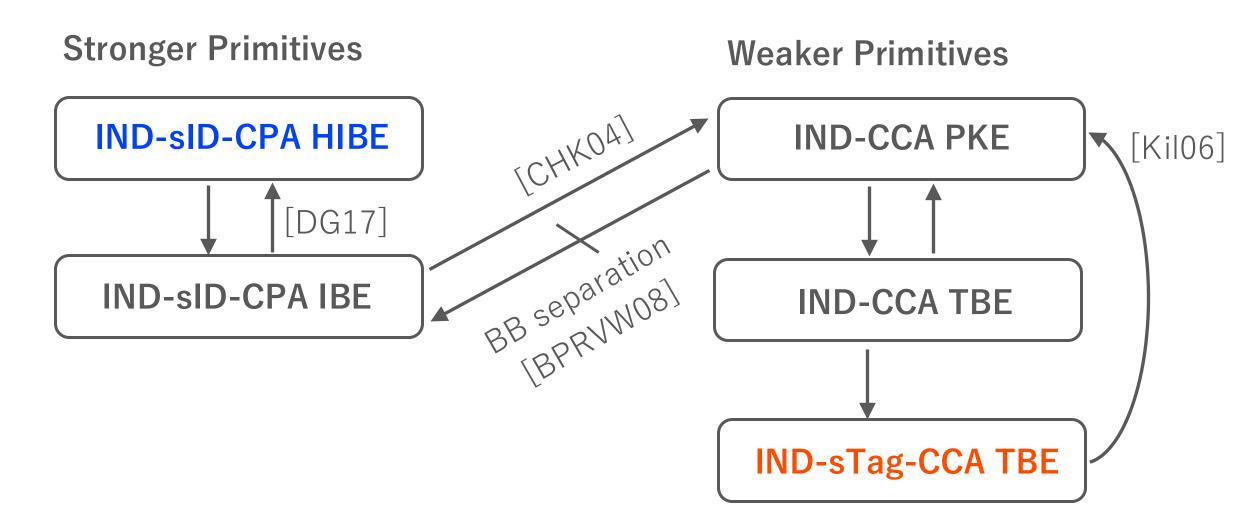
weaker primitive then HIBE without the ROM?

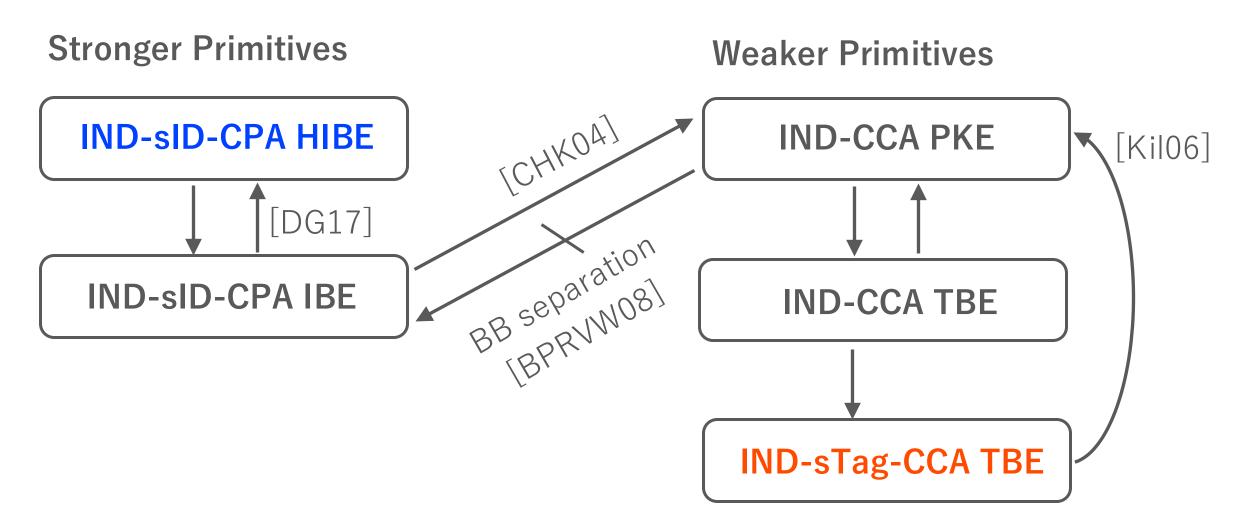
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Our Scheme	IND-sTag-CCA TBE + OTS + OW&CR Hash	Yes











TBE is weaker than HIBE.

# Definition of PKEET and its Security

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PKEET = (Setup, KGen, Enc, Dec, TDGen, Test)

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$$1^{\lambda} - Setup \rightarrow pp$$

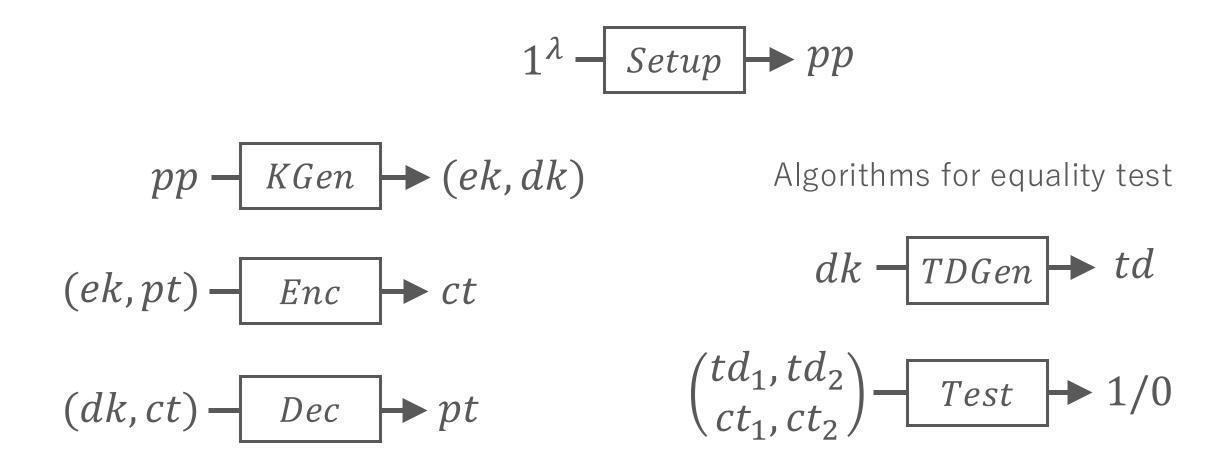
$$pp - KGen \rightarrow (ek, dk)$$

$$(ek, pt) - Enc \rightarrow ct$$

$$(dk, ct) - Dec \rightarrow pt$$

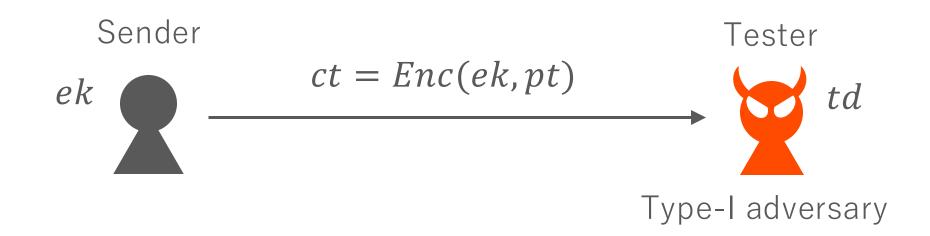
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#### Security of PKEET (OW-CCA against Type-I Adversary)

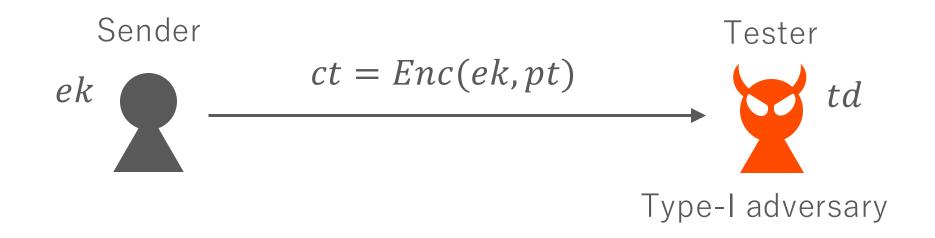
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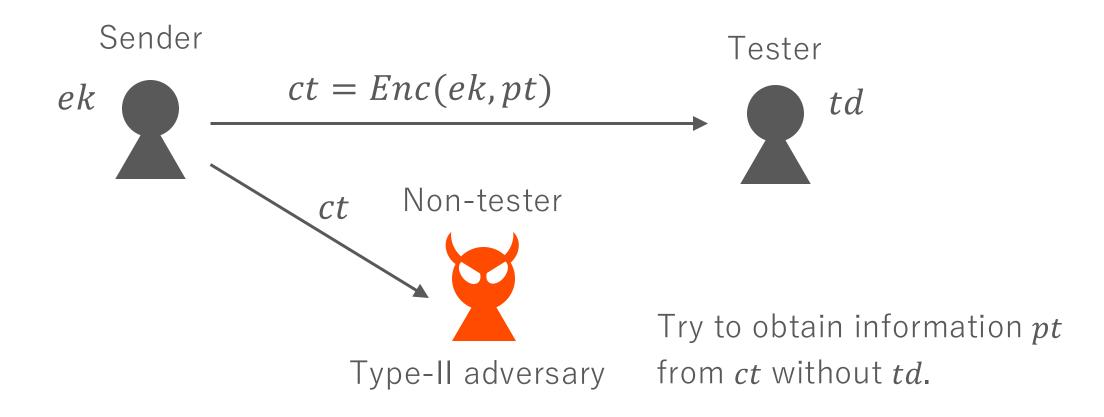


Try to obtain information *pt* from *ct* by using *td*.

The IND security is impossible for type-I adversaries. Instead, we consider the **OW-CCA** security for **type-I adversaries**.

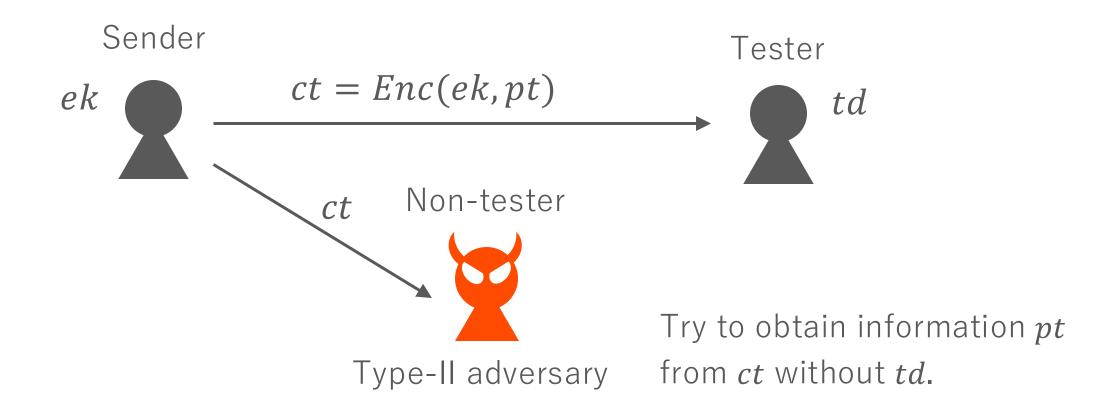
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Type-II adversary (non-tester) does have a trapdoor td.



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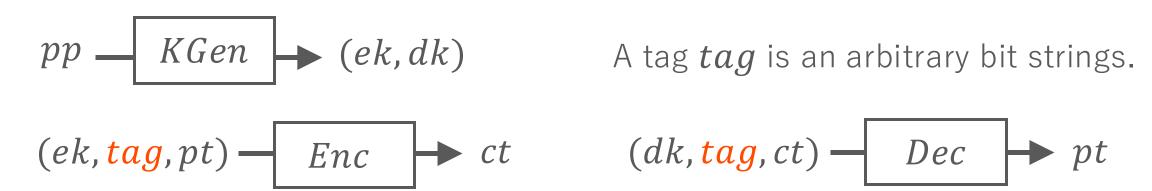
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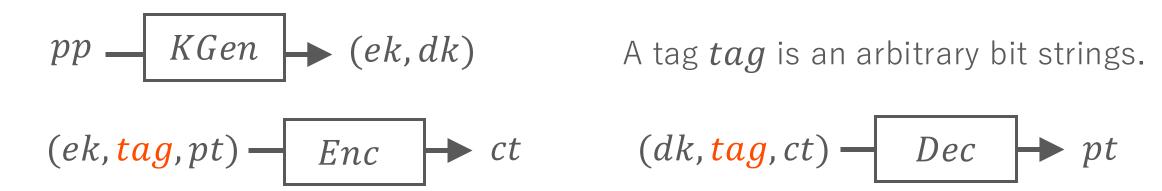
For type-II adversaries, we consider the IND-CCA security.

# How to Obtain Our Construction

#### Key Primitive: Tag-Based Encryption (TBE) [Kil06]



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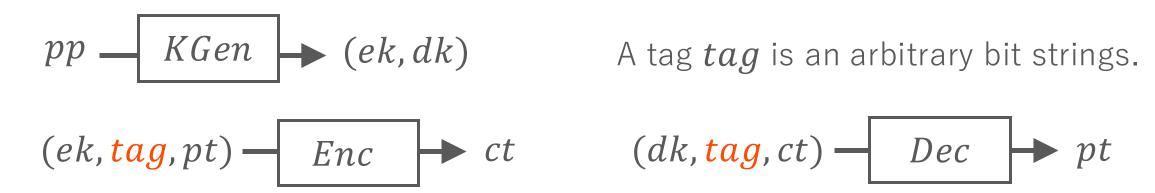


#### Correctness:

A ciphertext ct generated by (ek, tag) can be decrypted by using (dk, tag).

$$Dec(dk, tag\ Enc(ek, tag, pt)) = pt$$

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#### Observation [Kil06]:

TBE is sufficient for applying CHK transformation.

#### Our Construction Approach

#### Based PKEET (Not CCA secure)

A receiver prepare two tuple of TBE keys  $(ek_1, dk_1), (ek_2, dk_2)$ .

A sender generates ciphertexts

 $ct_1 \leftarrow TBE.Enc(ek_1, tag, pt), ct_2 \leftarrow TBE.Enc(ek_2, tag, H(pt))$ 

A trapdoor for equality tests is  $dk_2$ 

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**CHK transformation [CHK04]** 

#### **PKEET Scheme with IND-CCA security**

pp = H : OW & CR hash OTS : One-time signature

```
PKEET.KGen(1^{\lambda}):
(ek_1, dk_1) \leftarrow TBE_1.KGen(1^{\lambda})
(ek_2, dk_2) \leftarrow TBE_2.KGen(1^{\lambda})
(ek, dk) \leftarrow ((ek_1, ek_2), (dk_1, dk_2))
```

pp = H : OW & CR hash OTS : One-time signature

# $PKEET.KGen(1^{\lambda}):$ $(ek_1, dk_1) \leftarrow TBE_1.KGen(1^{\lambda})$ $(ek_2, dk_2) \leftarrow TBE_2.KGen(1^{\lambda})$ $(ek, dk) \leftarrow ((ek_1, ek_2), (dk_1, dk_2))$

```
PKEET.Enc(ek = (ek_1, ek_2), pt):
  (vk, sk) \leftarrow OTS.KGen(1^{\lambda})
  ct_1 \leftarrow TBE_1.Enc(ek_1, tag = vk, pt)
  ct_2 \leftarrow TBE_2. Enc(ek_2, tag = vk, H(pt))
  \sigma \leftarrow OTS.Sign(sk,(ct_1,ct_2))
  ct \leftarrow (vk, ct_1, ct_2, \sigma)
```

pp = H : OW & CR hash OTS : One-time signature

```
PKEET.Dec(dk = (dk_1, dk_2), ct = (vk, ct_1, ct_2, \sigma)):
OTS.Ver(vk, (ct_1, ct_2), \sigma) = 1 ?
pt \leftarrow TBE_1.Dec(dk_1, tag = vk, ct_1),
h \leftarrow TBE_2.Dec(dk_2, tag = vk, ct_2),
If H(pt) = h \text{ return } pt.
Otherwise \text{ return } \bot.
```

$$PKEET.TGen(dk = (dk_1, dk_2)):$$

$$td = dk_2$$

$$PKEET.TGen(dk = (dk_1, dk_2)): \\
 td = dk_2$$

```
PKEET.Test \begin{pmatrix} td = dk_2, td' = dk'_2, \\ ct = (vk, ct_1, ct_2, \sigma), ct' = (vk', ct'_1, ct'_2, \sigma') \end{pmatrix}:h \leftarrow TBE_2.Dec(dk_2, tag = vk, ct_2), h' \leftarrow TBE_2.Dec(dk'_2, tag' = vk', ct'_2), If h = h' return 1.
```

Otherwise return 0.

#### Conclusion

We give a generic construction of PKEET.

Our construction is based on TBE.

The security is proven without the ROM.

Future work

Generic construction of PKEET with shorter ciphertext size in the standard model.

#### Thank you for listening!

# Appendix

