

# Improved Security Proof for the Camenisch-Lysyanskaya Signature-Based Synchronized Aggregate Signature Scheme

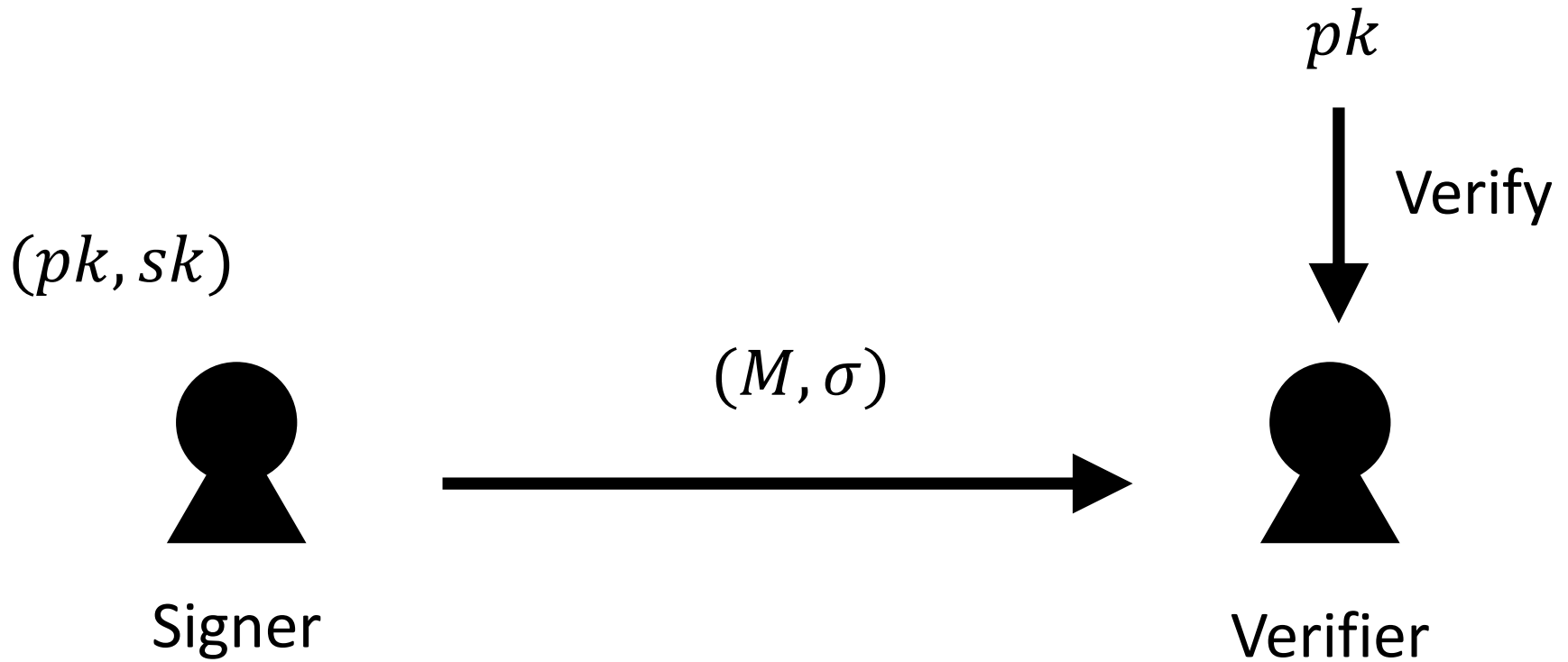
Masayuki Tezuka    Keisuke Tanaka

Tokyo Institute of Technology

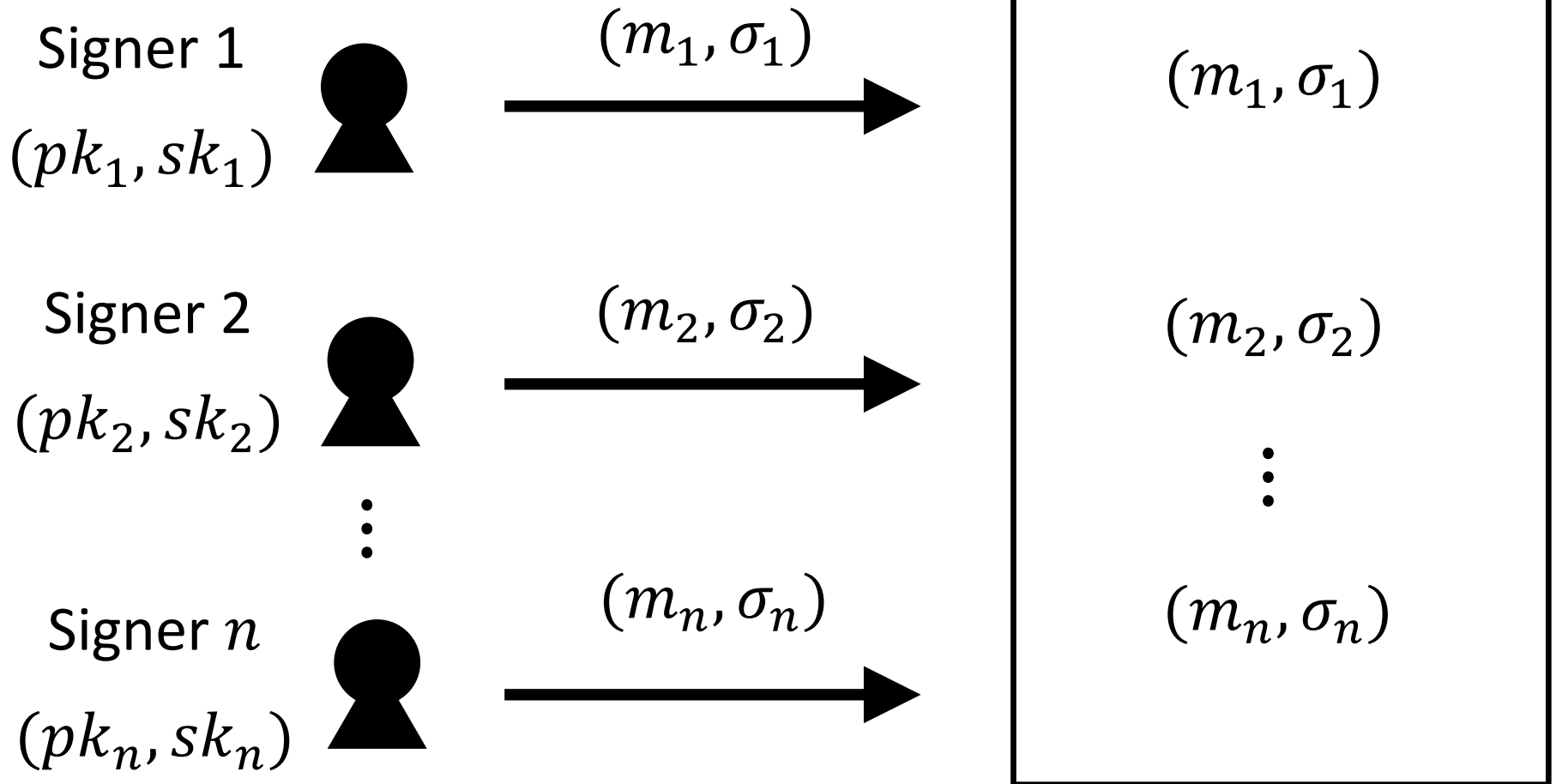
Version: 2020/12/23

ACISP 2020 Full presentation slide

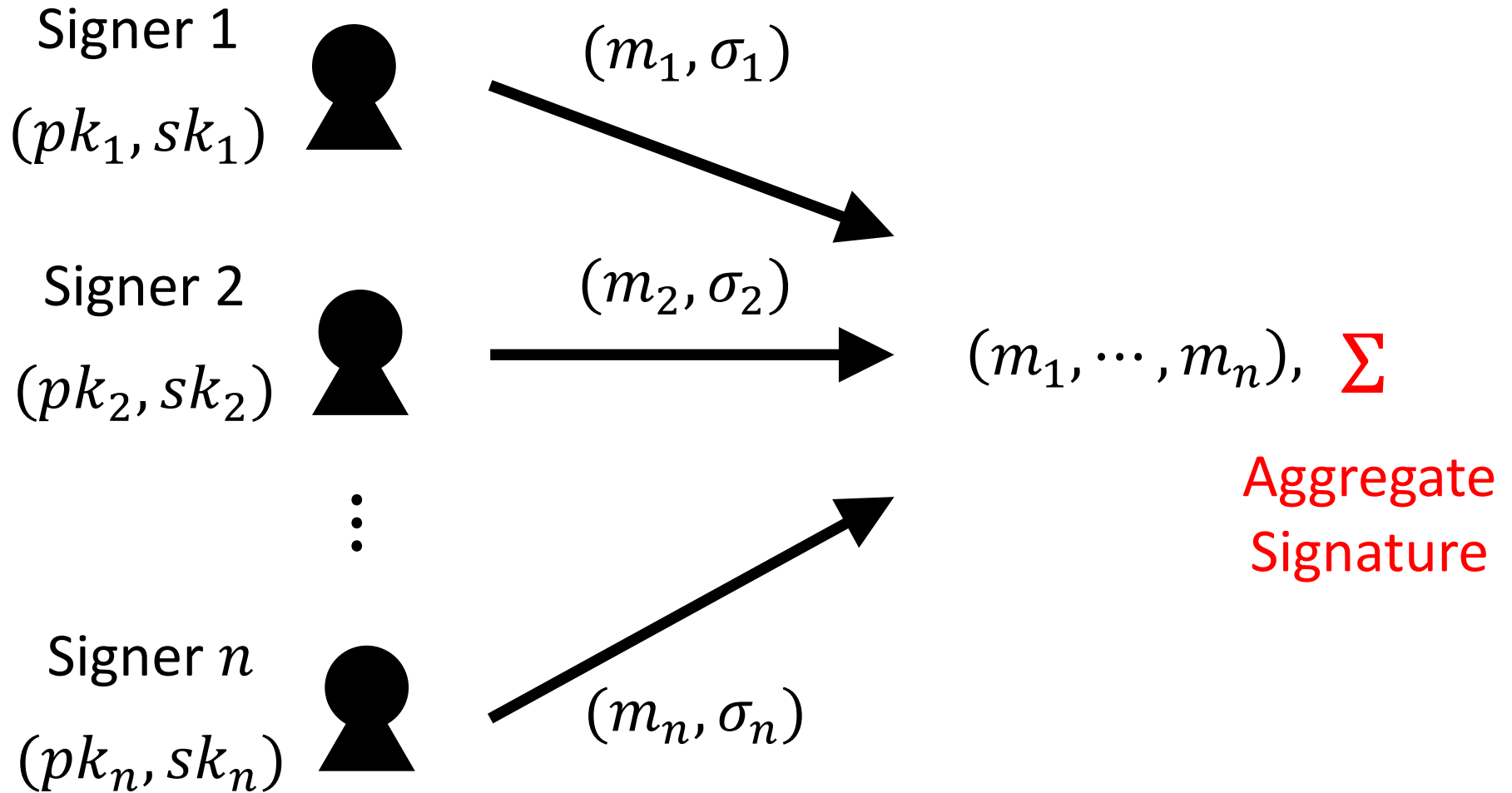
# Digital Signature



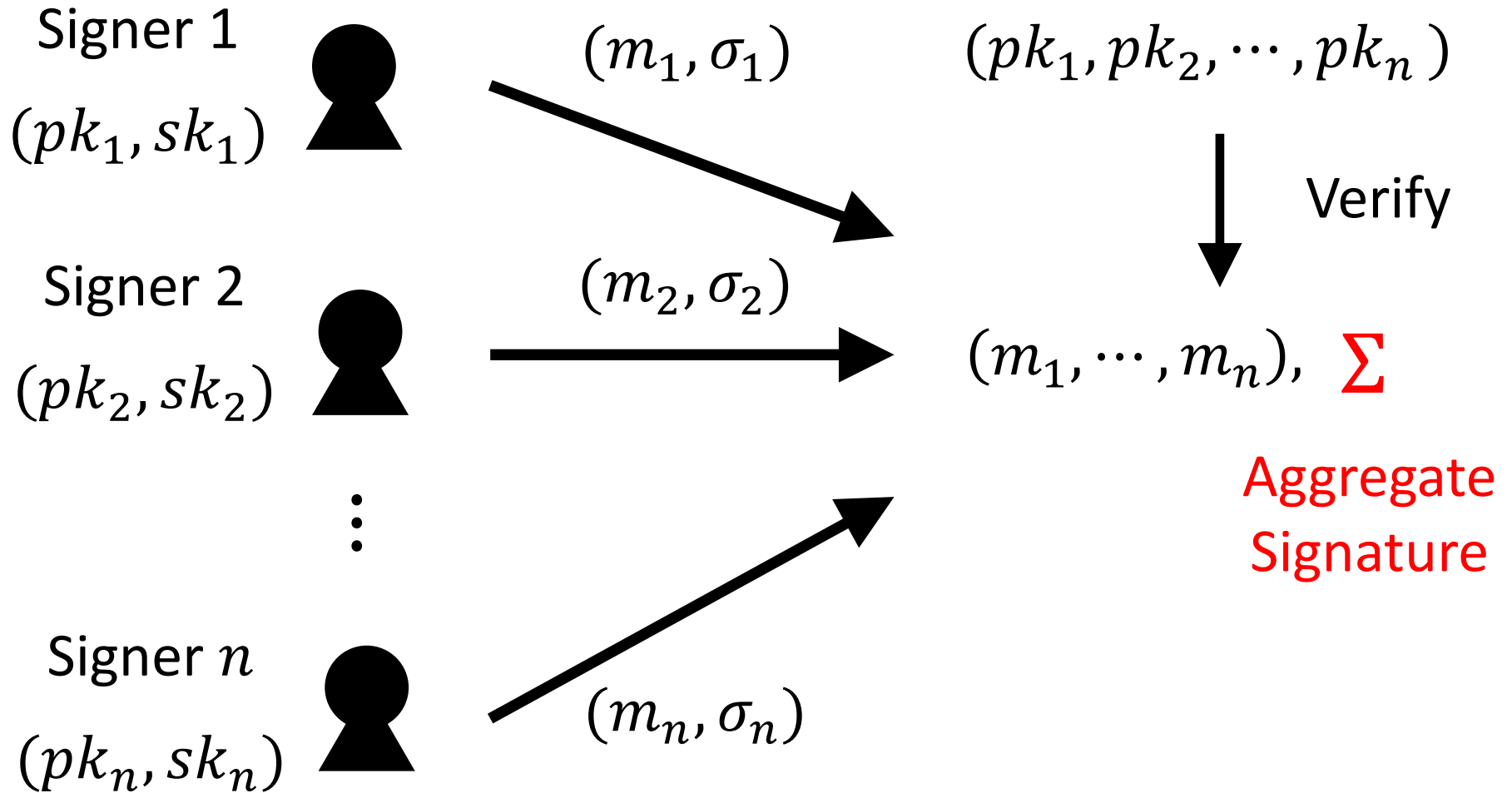
# Aggregate Signature



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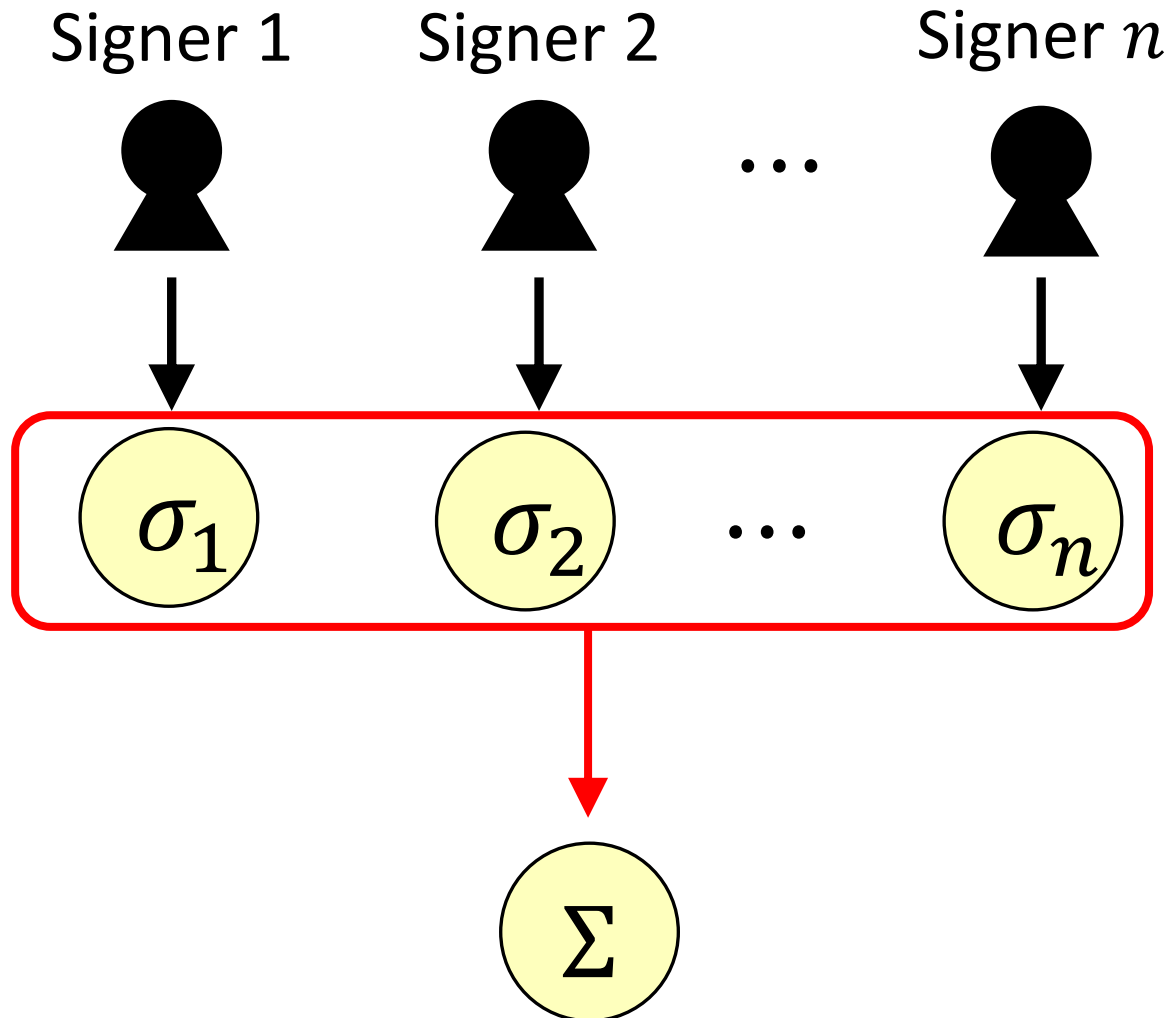
# Types of Aggregate Signature

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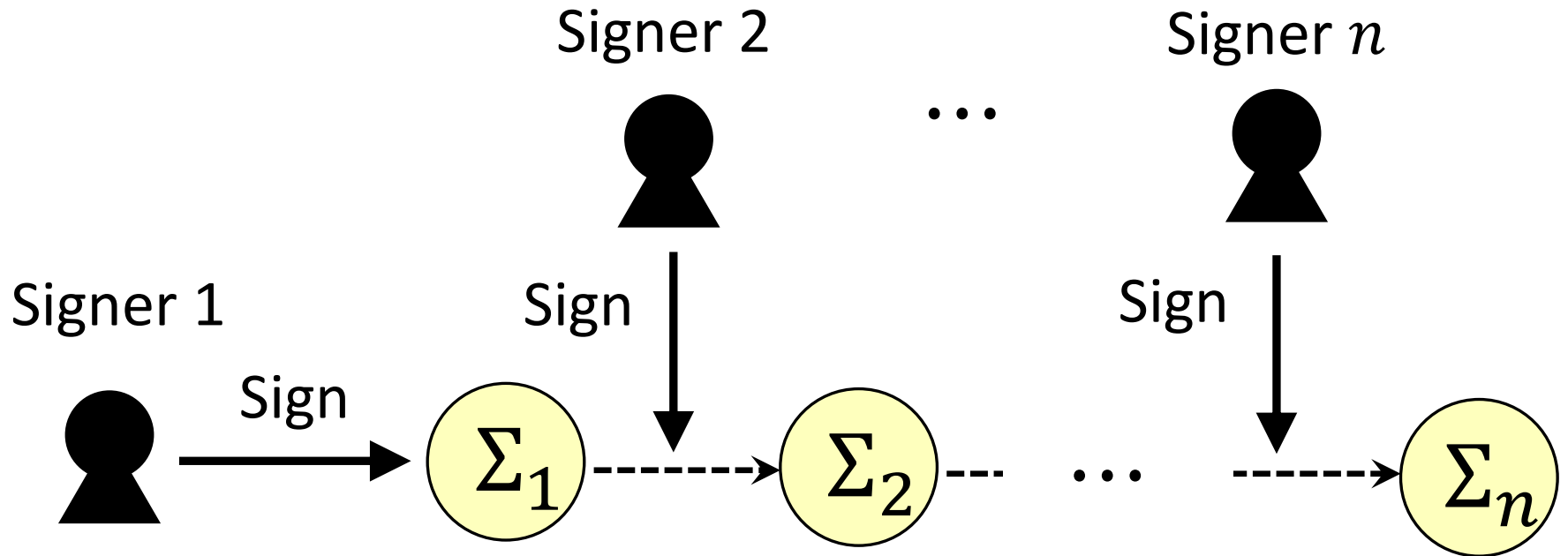
- Full aggregate signature [BGLS03]
- Sequential aggregate signature (SeqAS)  
[LMRS04]
- Synchronized aggregate signature (SyncAS)  
[GR06,AGH10]

etc...

# Full Aggregate Signature [BGLS03]



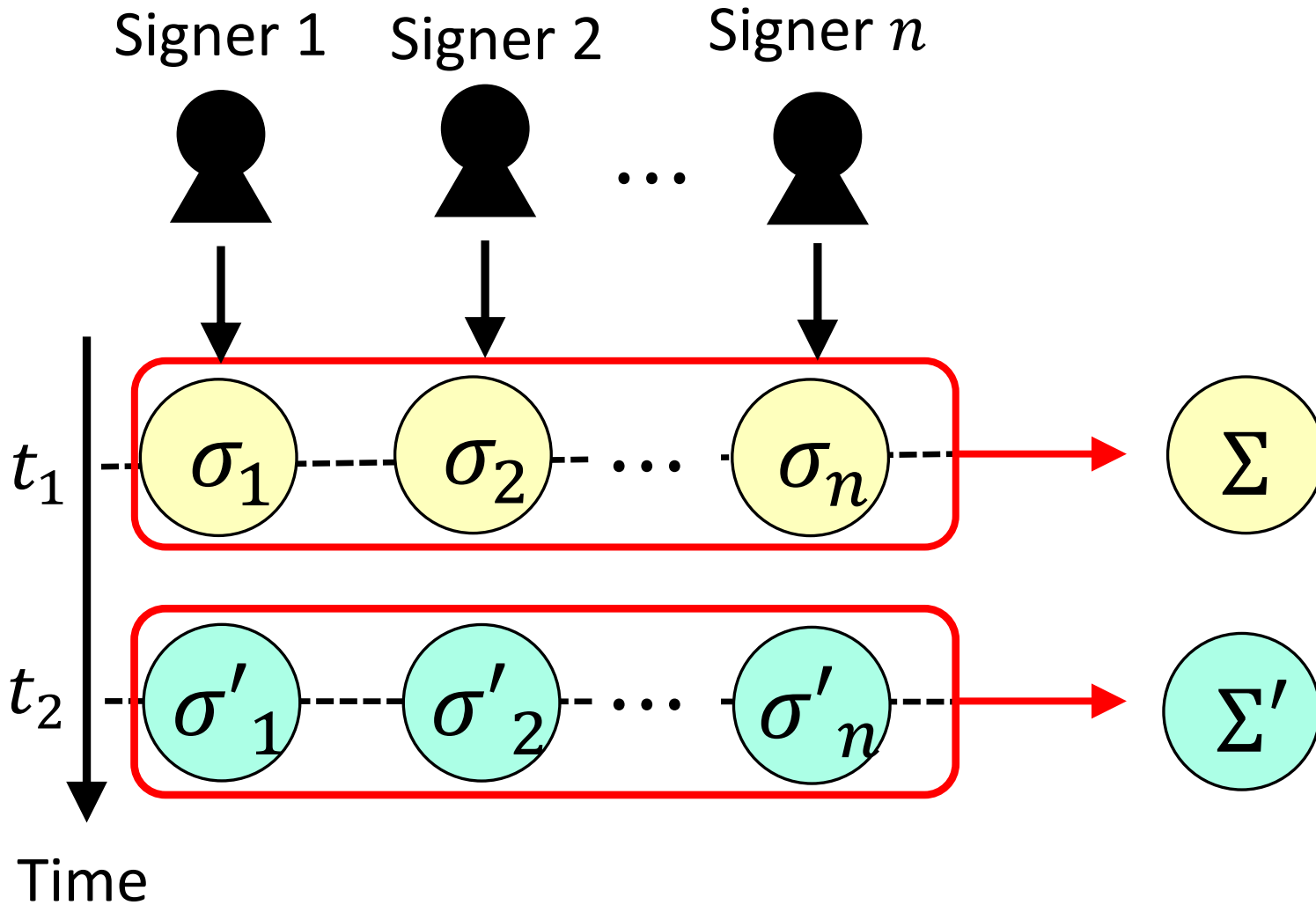
# Sequential Aggregate Signature (SeqAS) [LMRS04]





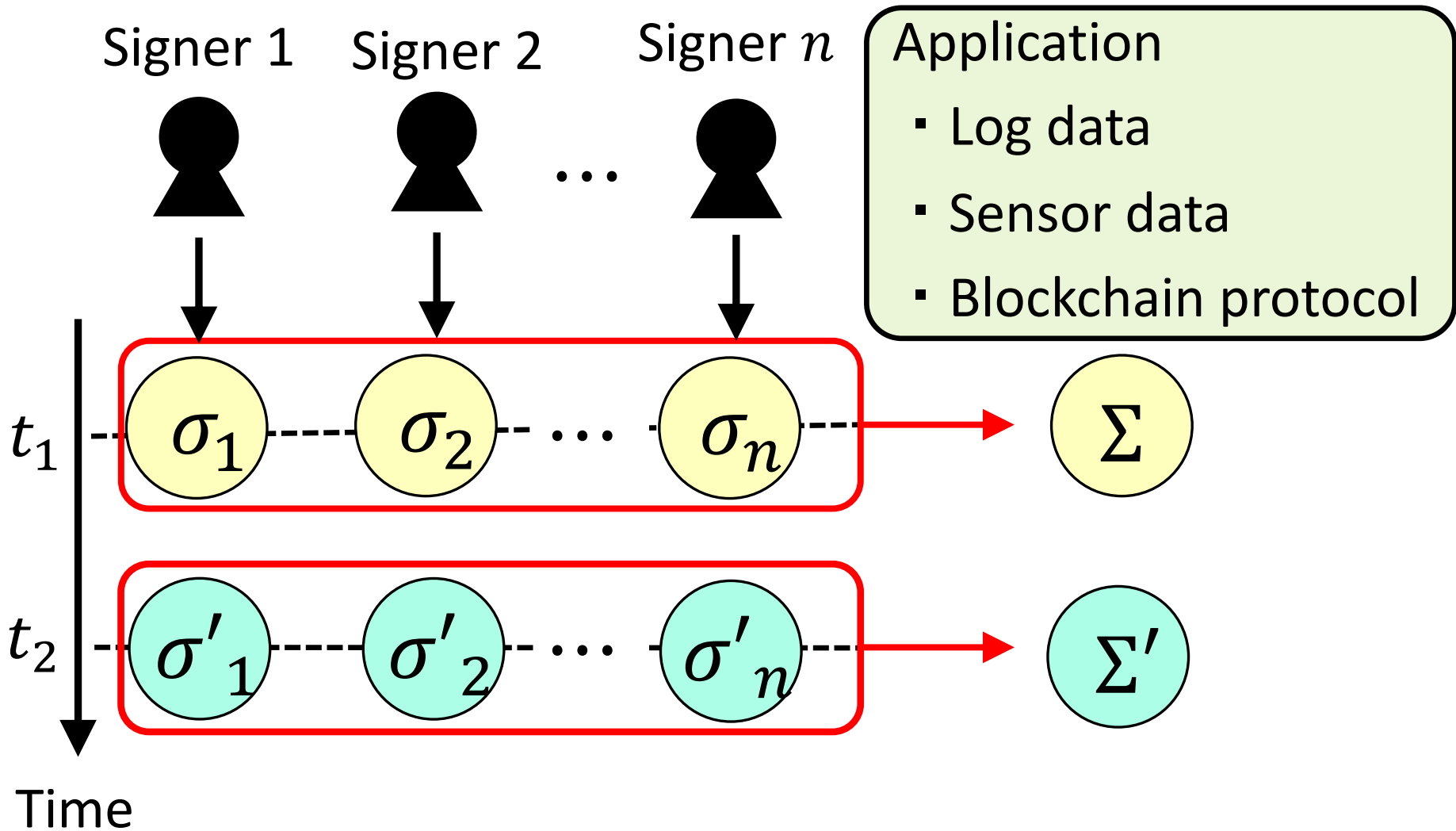
# Synchronized Aggregate Signature (SyncAS)

[GR06,AGH10]



# Synchronized Aggregate Signature (SyncAS)

[GR06,AGH10]



# Syntax of SyncAS [AGH 10]

$\text{Setup}(1^\lambda, 1^T) \rightarrow pp$

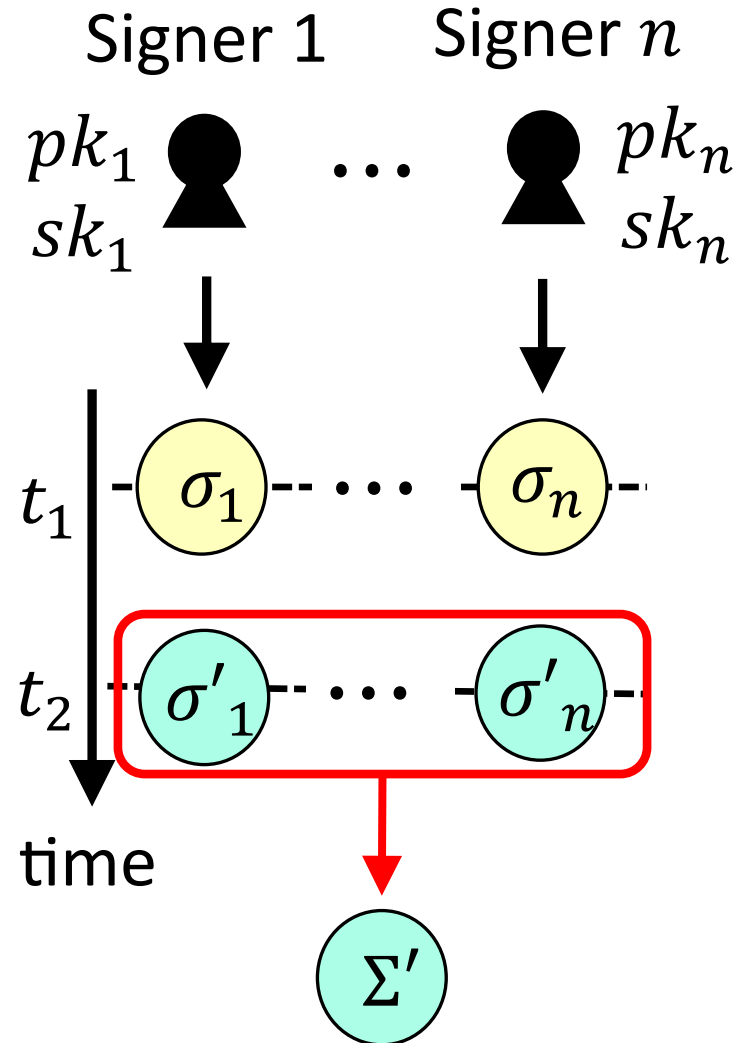
$\text{KeyGen}(pp) \rightarrow (pk, sk)$

$\text{Sign}(sk, t, m) \rightarrow \sigma$

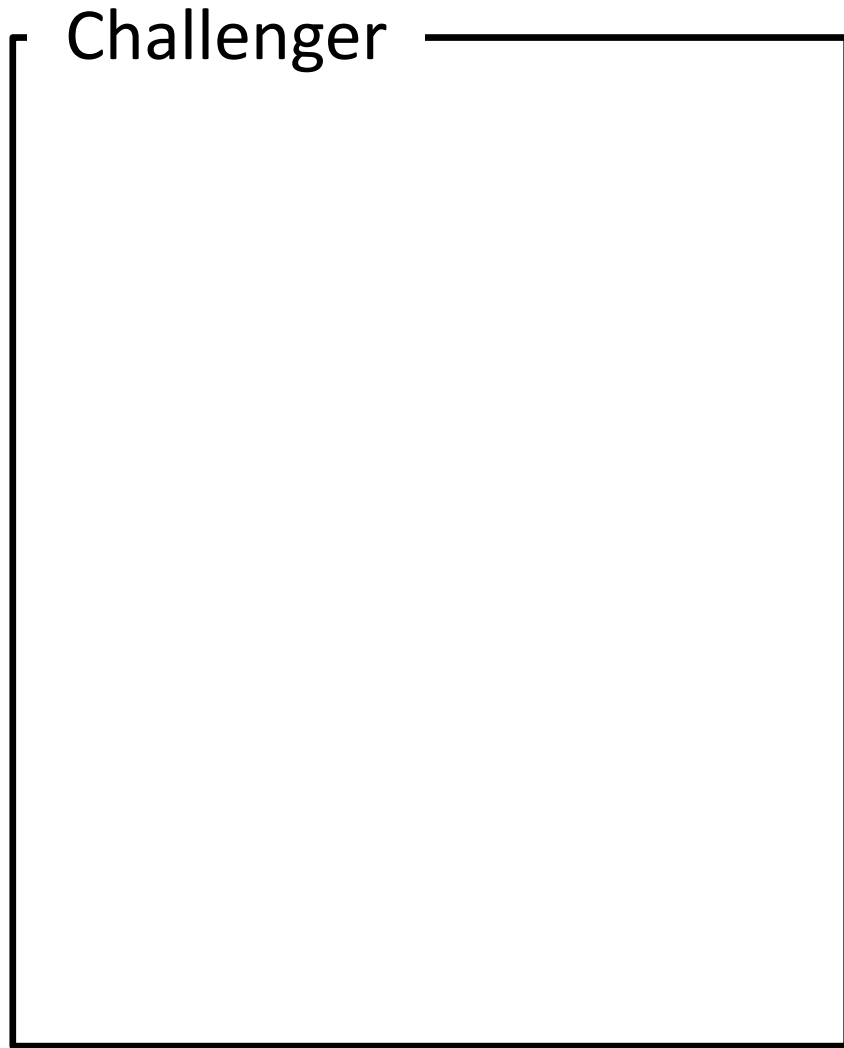
$\text{Verify}(pk, m, \sigma) \rightarrow 0 \text{ or } 1$

$\text{Aggregate}(\{pk_i, m_i, \sigma_i\}_{i=1}^n) \rightarrow \Sigma$

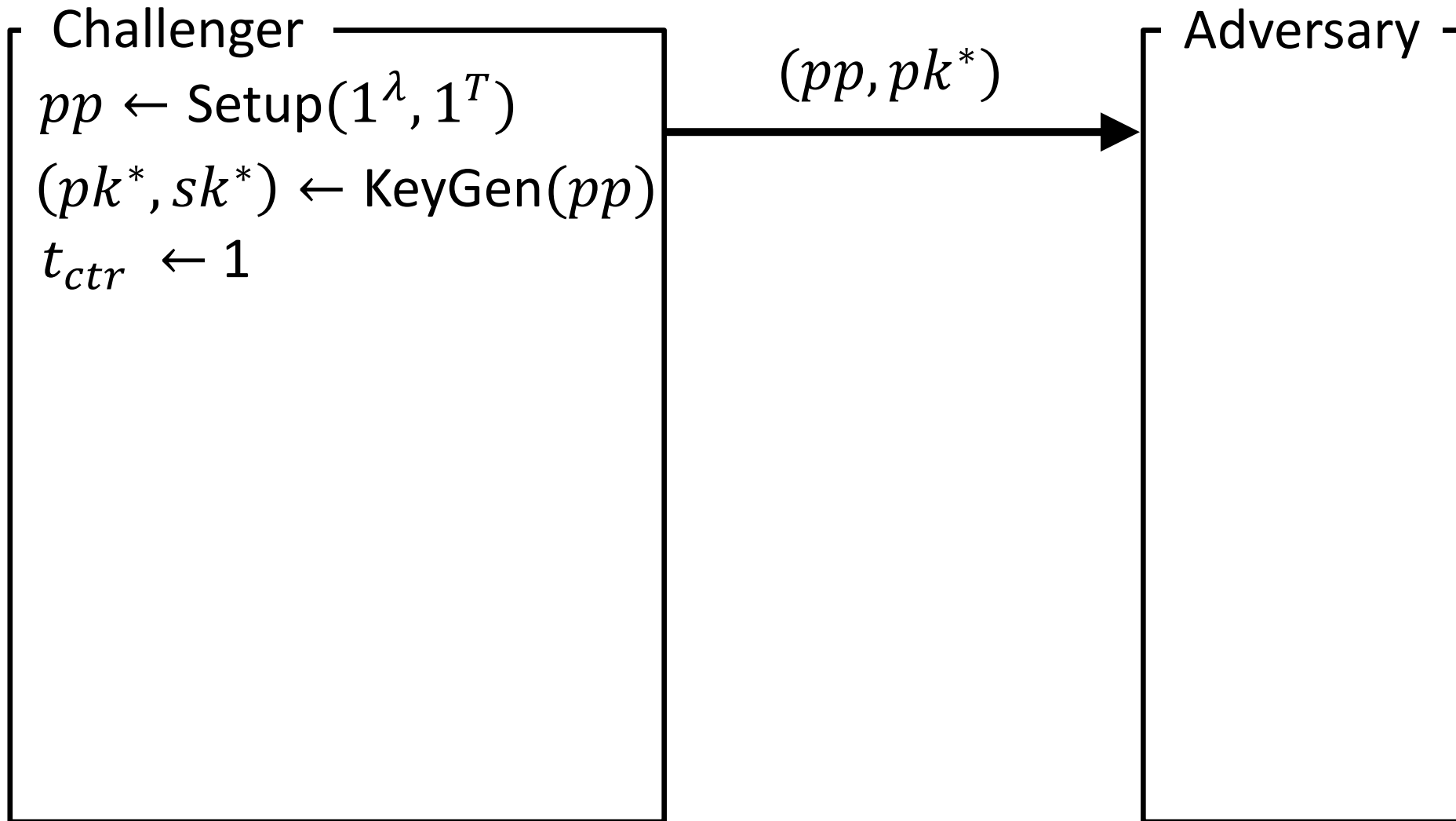
$\text{AggVerify}(\{pk_i, m_i\}_{i=1}^n, \Sigma) \rightarrow 0 \text{ or } 1$



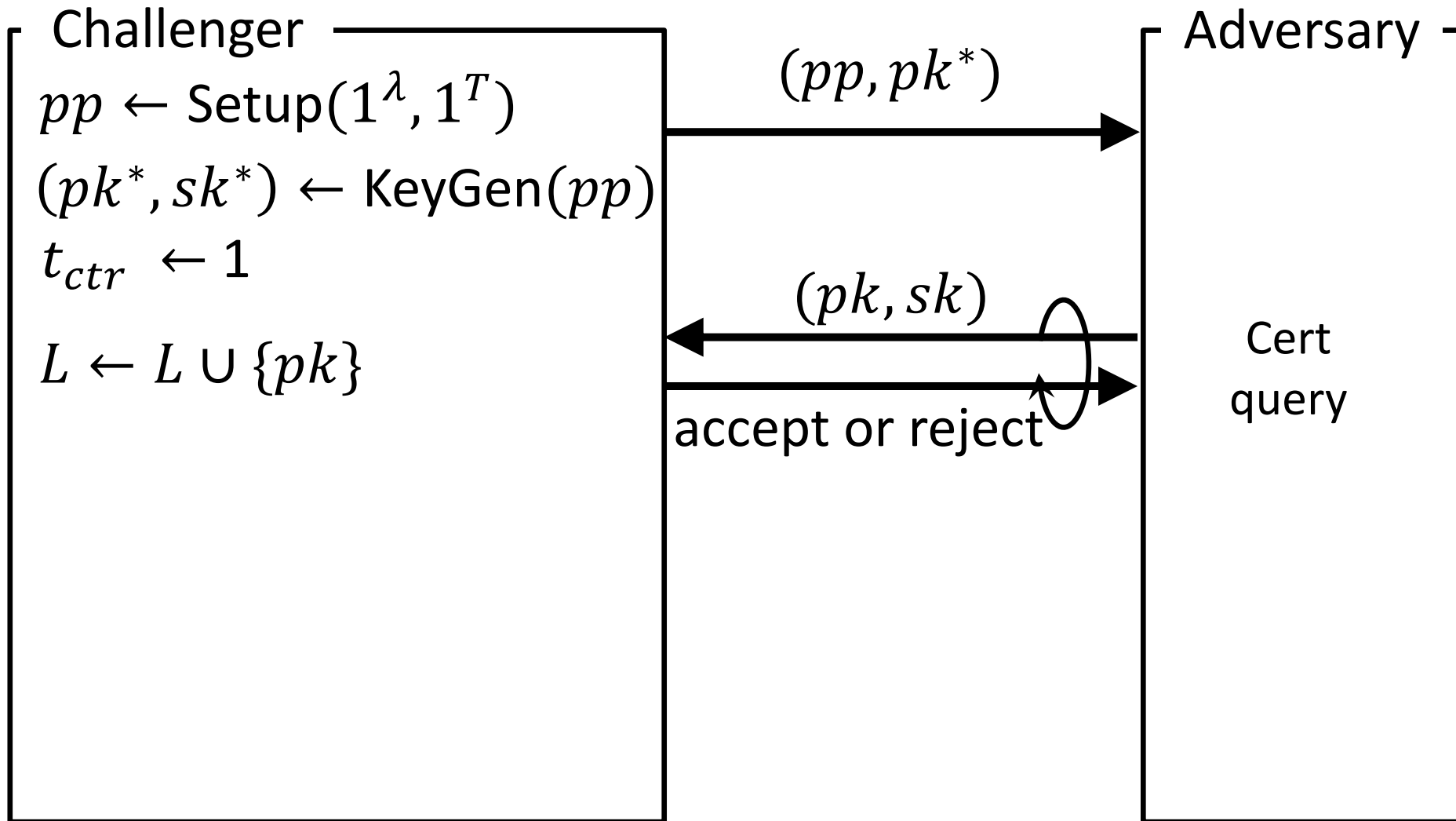
# EUF-CMA Security for SyncAS in the Certified-Key Model [AGH 10]



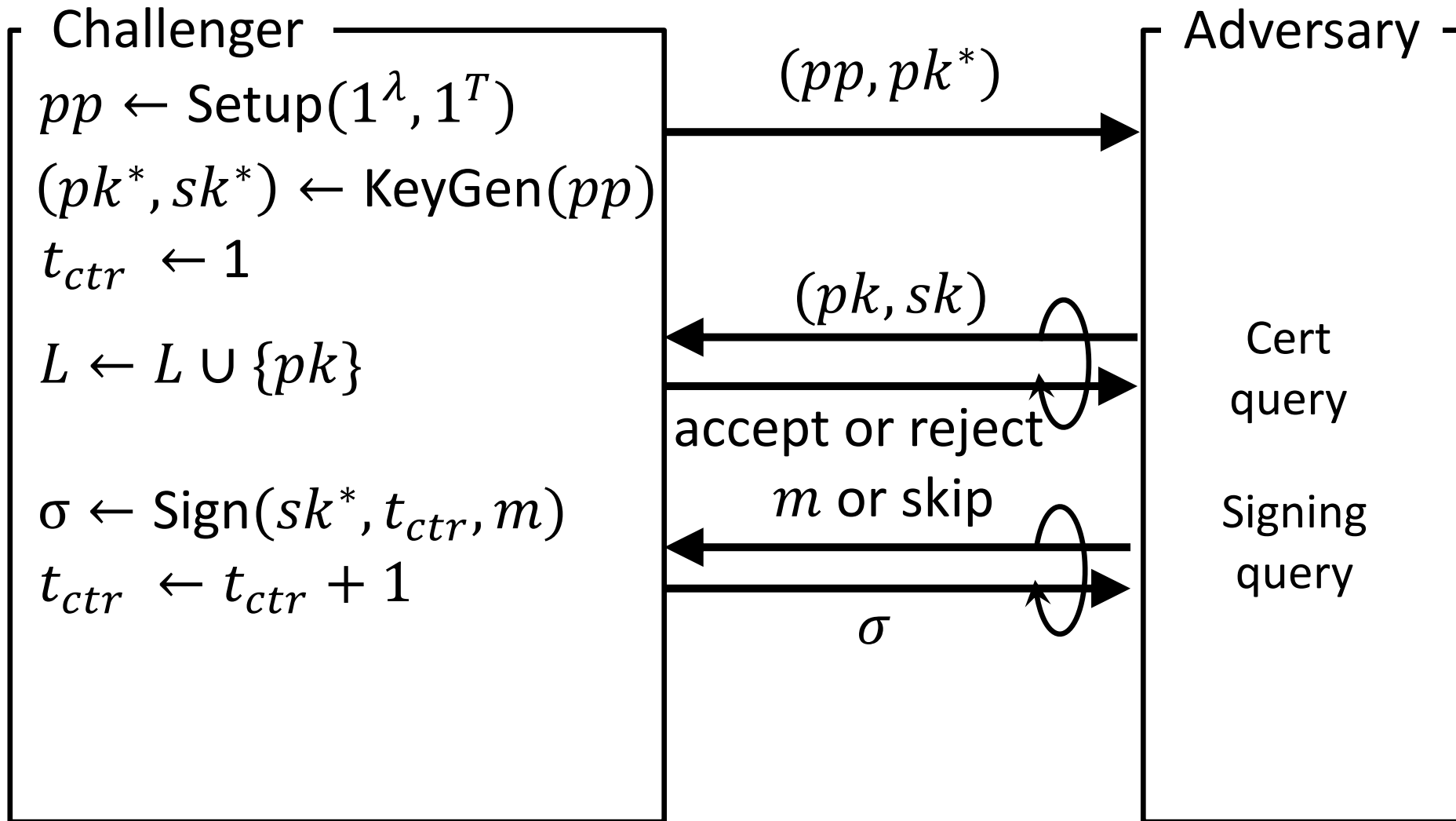
# EUF-CMA Security for SyncAS in the Certified-Key Model [AGH 10]



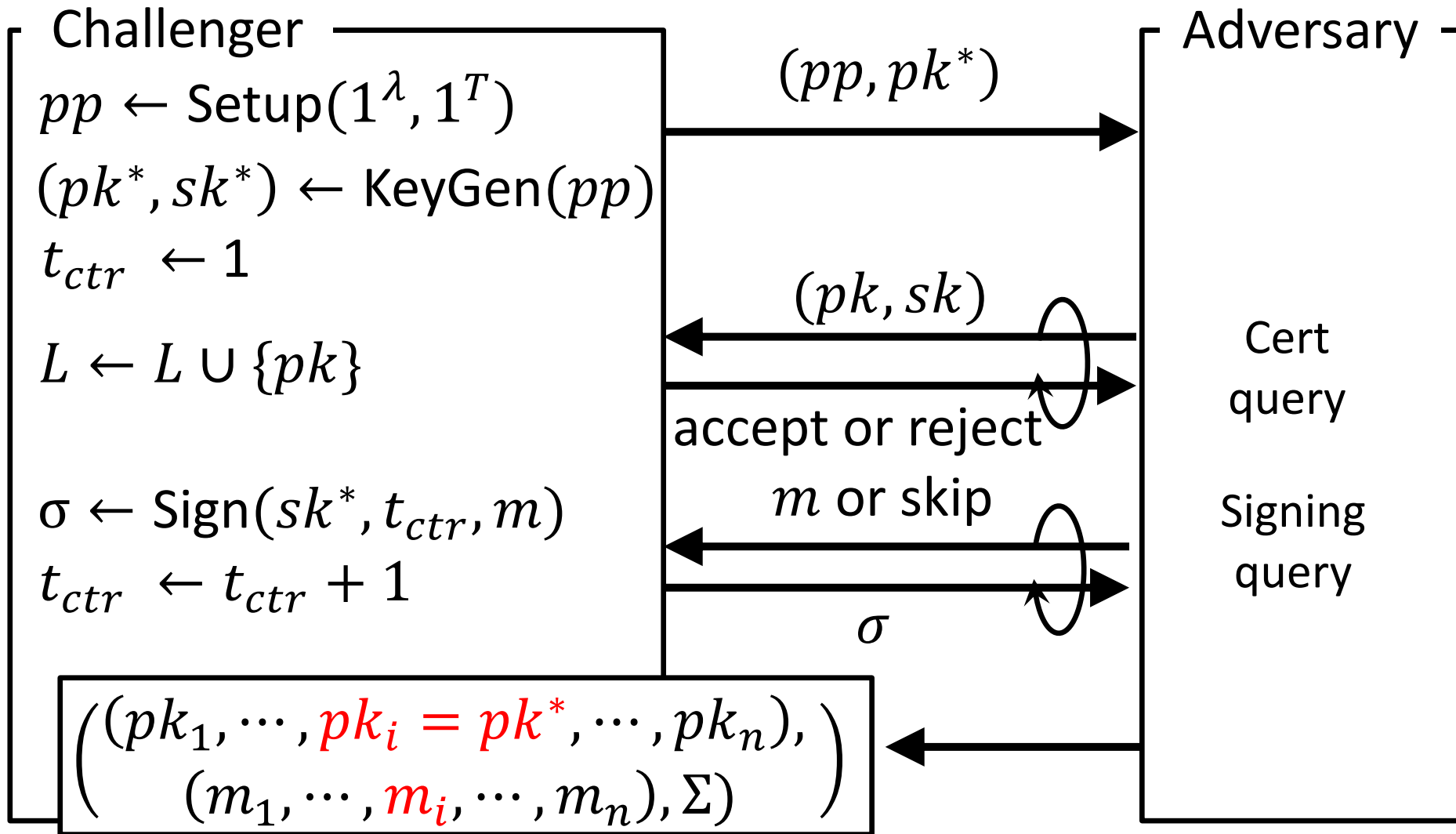
# EUF-CMA Security for SyncAS in the Certified-Key Model [AGH 10]



# EUFCMA Security for SyncAS in the Certified-Key Model [AGH 10]



# EUFCMA Security for SyncAS in the Certified-Key Model [AGH 10]





# EUF-CMA Security for SyncAS in the Certified-Key Model [AGH 10]

The adversary wins if:

1.  $\left( \begin{array}{l} (pk_1, \dots, pk_i = pk^*, \dots, pk_n), \\ (m_1, \dots, m_i, \dots, m_n), \Sigma \end{array} \right)$  is valid.
2. Never queried  $m_i$  for the signing oracle.
3. All keys  $(pk_1, \dots, pk_n)$  are registered.

# SyncAS Based on Bilinear Group

Scheme	Assumption	pk size	Agg sig size	Agg Ver (in parinig)
GR06	CDH + ROM	ID	3	3
AGH10	CDH	1	3	$k + 3$
AGH10	CDH + ROM	1	2	4
LLY13	OT-LRSW + ROM	1	2	3



Most efficient  
scheme

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Most efficient scheme



Interactive assumption !

# Non-Interactive Assumption

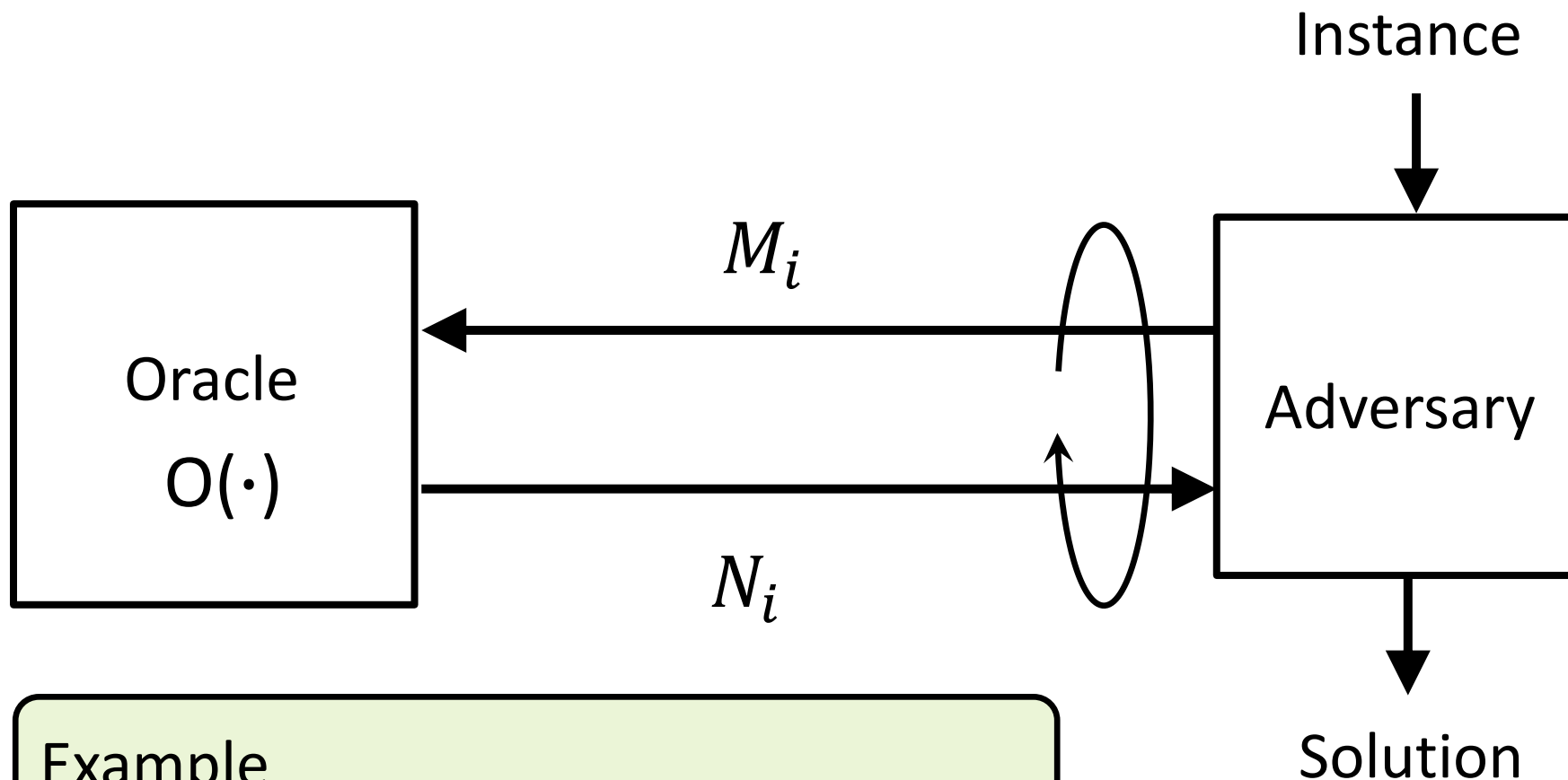


## Example

DDH, CDH, DL,  
SXDH, qSDH

etc...

# Interactive Assumption

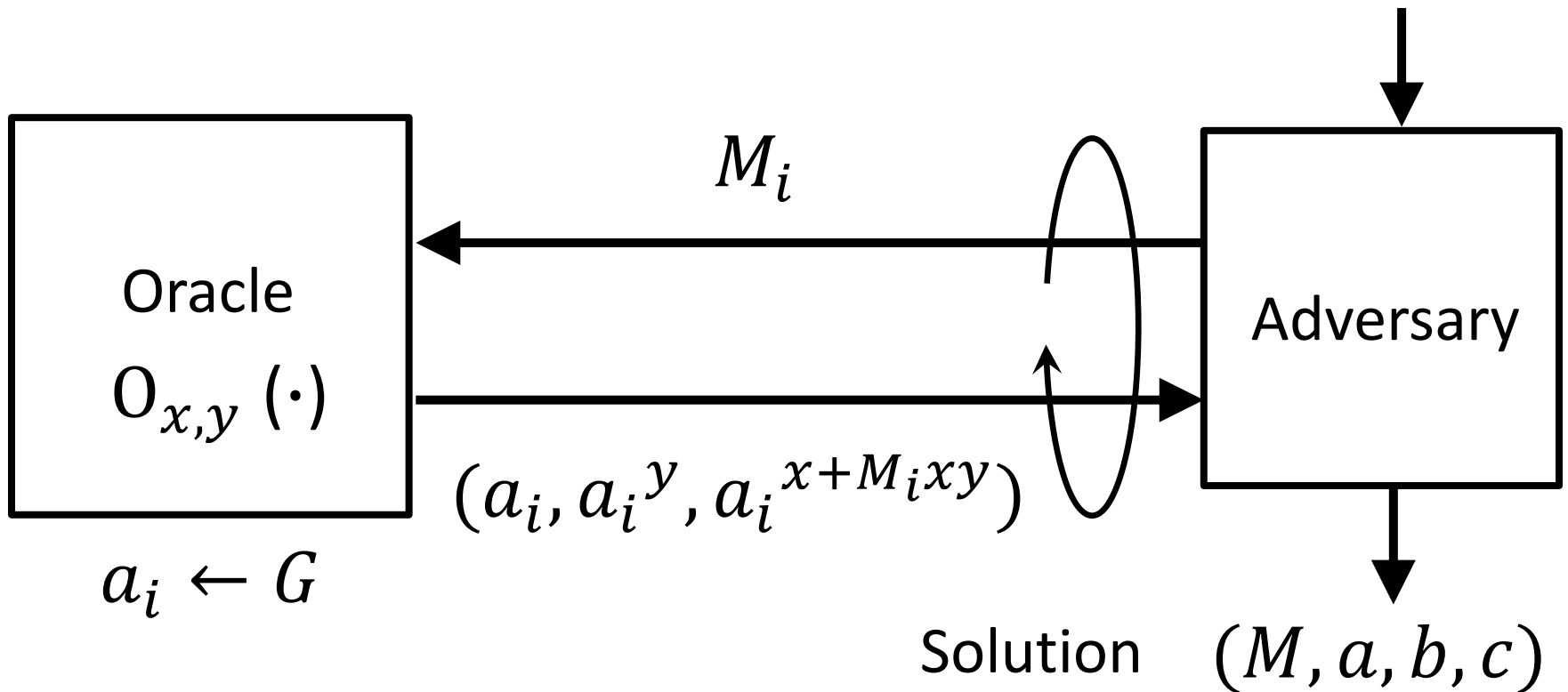


Example

LRSW [LRSW99], PS [PS16] etc...

# (OT-)LRSW Assumption [LRSW99]

Instance  $(p, G, G_T, e, g, X = g^x, Y = g^y)$



$M \notin \{M_i\}, M \in \mathbb{Z}_p^*, a \in G, b = a^y, c = a^{x+Mxy}$

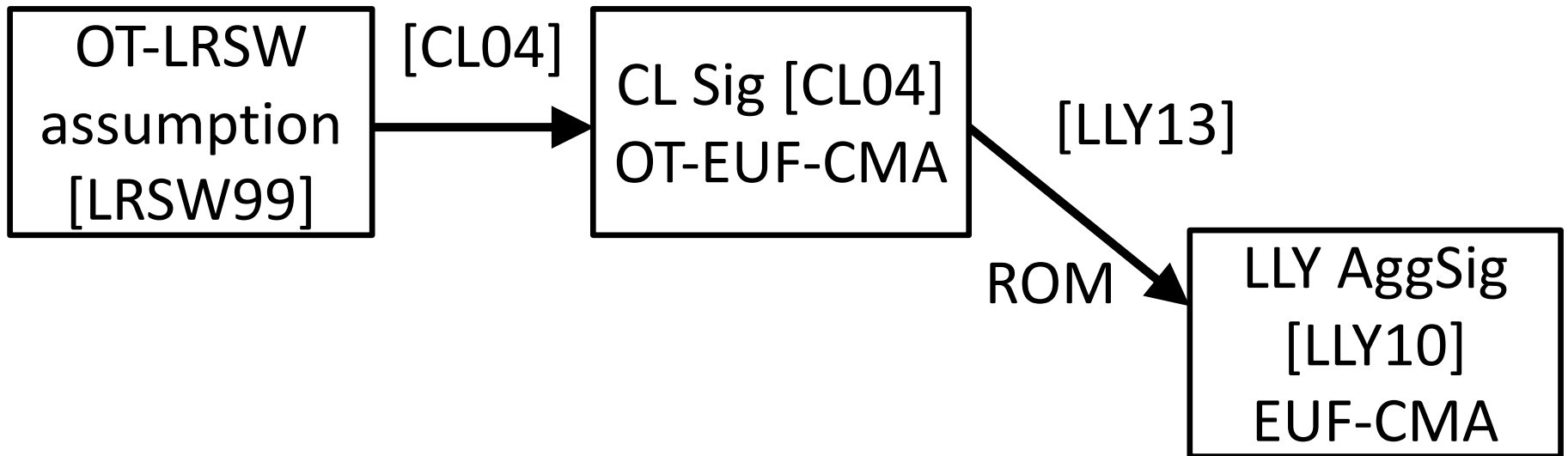
# Our Contribution

Scheme	Assumption	pk size	Agg sig size	Agg Ver (parinig)
GR06	CDH + ROM	ID	3	3
AGH10	CDH	1	3	$k + 3$
AGH10	CDH + ROM	1	2	4
LLY13	OT-LRSW + ROM	1	2	3
LLY13 (New Proof)	1-MSDH-2 + ROM			



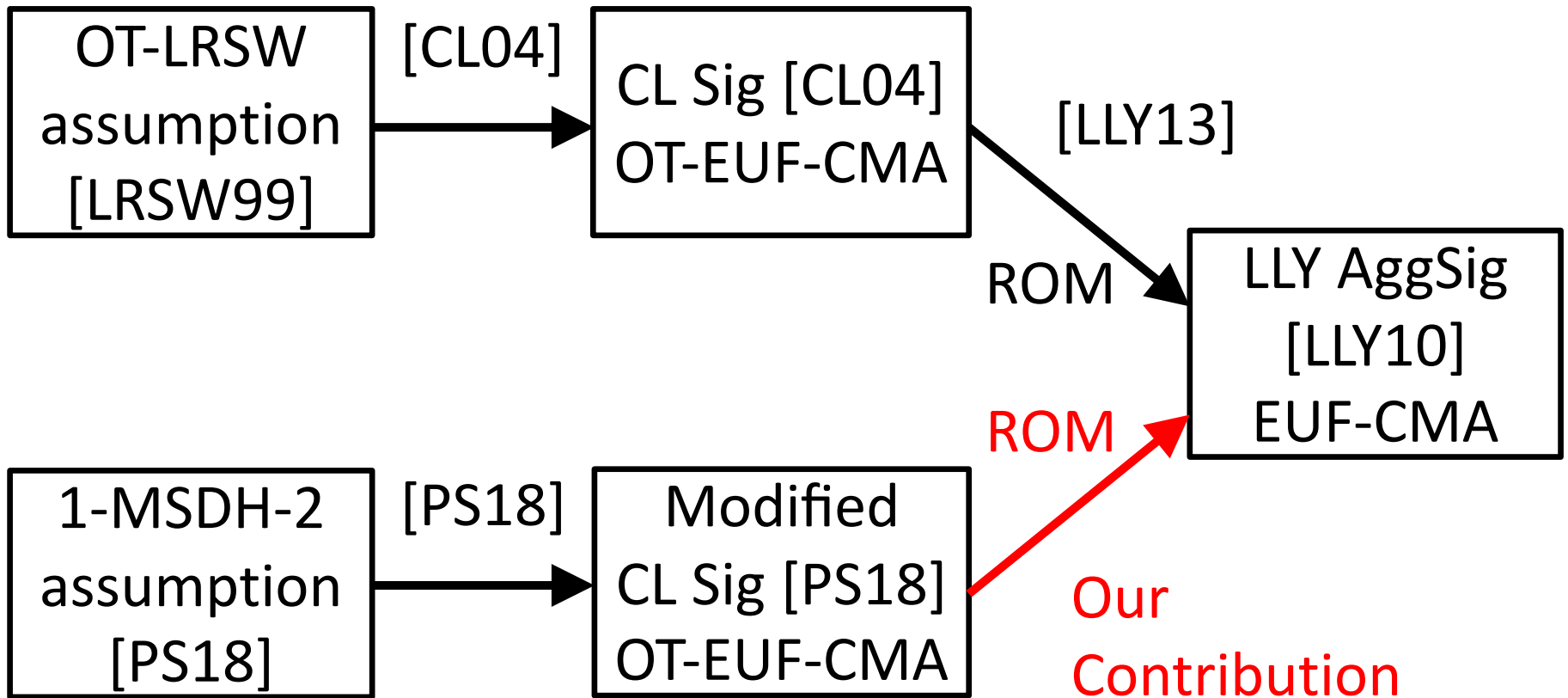
Non-interactive  
and static  
assumption !

# Security proof for LLY SyncAS

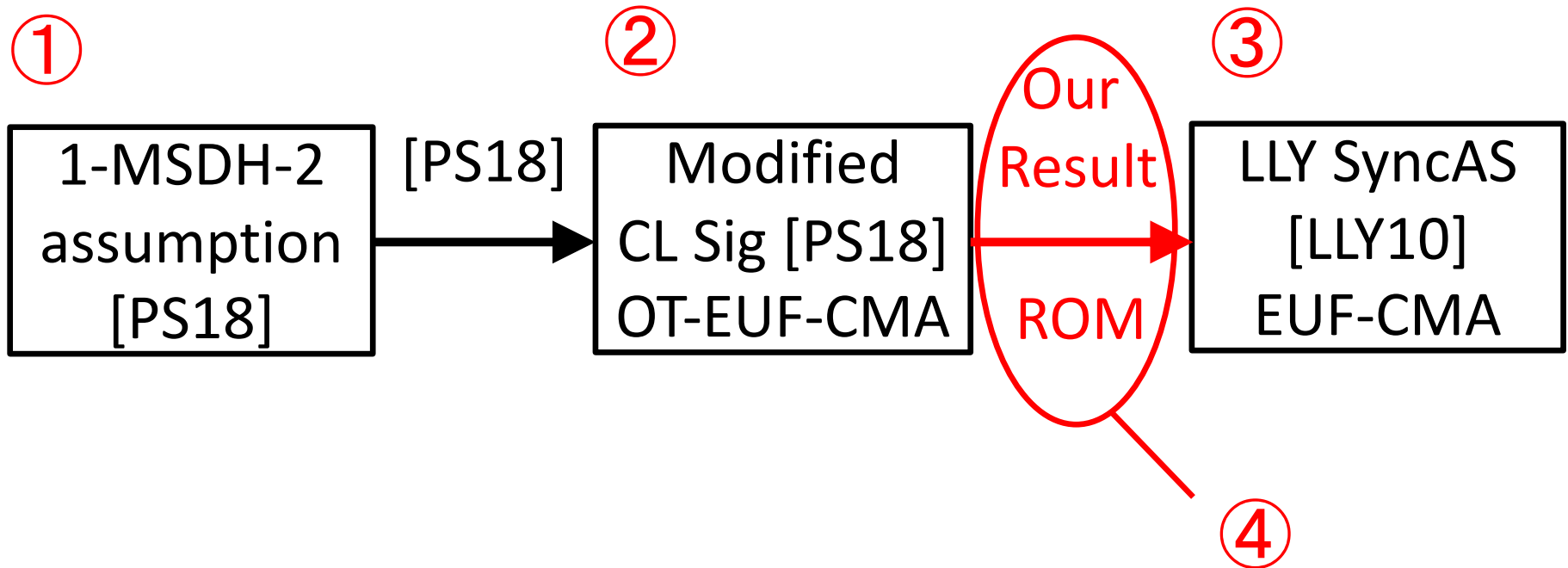




# Security proof for LLY SyncAS



# Our Contribution



# ① $q$ -MSDH-2 Assumption [PS18]

Instance  $\left( p, G, G_T, e, g, \left\{ g^{x^i}, g^{b \cdot x^i} \right\}_{i=1}^{q+1}, g^a, g^{abx} \right)$



If we fix  $q = 1$ ,  
we can regard 1-MSDH-2  
as a static assumption.

Solution  $\left( w, P, h^{\frac{1}{x+w}}, h^{\frac{a}{x \cdot P(x)}} \right)$

$w \neq 0, \deg(P) \leq q,$

$X + w$  and  $P(X)$  are relative prime.

## ② Modified CL Signature (MCL Sig) [PS18]

$$pp = (p, G, G_T, e)$$

KeyGen( $pp$ )

$$sk = (x, y, z) \leftarrow Z_p^3$$

$$g \leftarrow G^*, X \leftarrow g^x,$$

$$Y \leftarrow g^y, Z \leftarrow g^z$$

$$pk = (g, X, Y, Z)$$

Verify( $pk, m, \sigma$ )

$$e(A, Y) = e(B, g) ?$$

$$e(A, Z) = e(C, g) ?$$

$$e(C, Y) = e(D, g) ?$$

$$e(AB^m D^{m'}, X) = e(E, g) ?$$

Sign( $sk, m \in Z_p$ )

$$m' \leftarrow Z_p, A \leftarrow G^*, B \leftarrow A^y$$

$$C \leftarrow A^z, D \leftarrow C^y, E \leftarrow A^x B^{mx} D^{m'x}$$

$$\sigma \leftarrow (m', A, B, C, D, E)$$

## ② Modified CL Signature (MCL Sig) [PS18]

Theorem [PS18]

If the  $q$ -MSDH-2 assumption holds,  
the modified CL signature is EUF-CMA secure.  
( $q$  is a bound on the number of signing queries. )

We only use

If the 1-MSDH-2 assumption holds,  
the modified CL signature is OT-EUF-CMA secure.

### ③ LLY SyncAS [LLY13]

$$pp = (p, G, G_T, e, g, H_1, H_2, H_3)$$

Public key and Secret key

$$sk_i = x_i \leftarrow Z_p, \quad pk_i = X_i \leftarrow g^{x_i}$$

Signature on a message  $m \in Z_p$  in time period  $t$

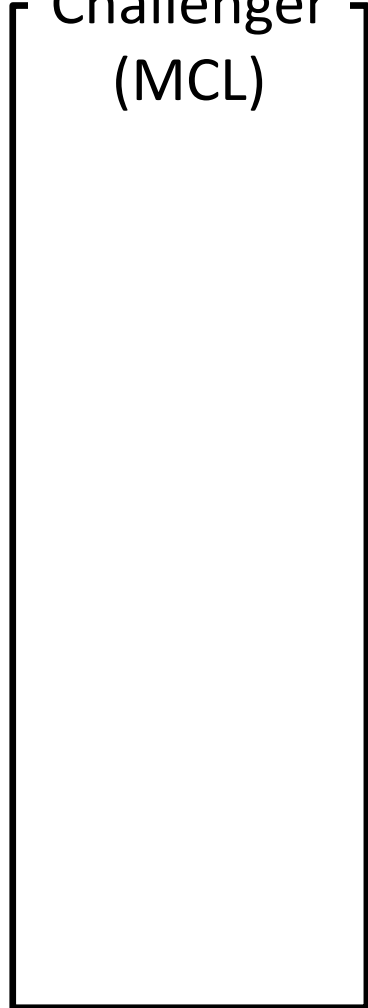
$$E_i \leftarrow H_1(t)^{x_i} \cdot H_2(t)^{H_3(t, m_i) x_i}, \sigma \leftarrow (E_i, t)$$

Aggregate signature

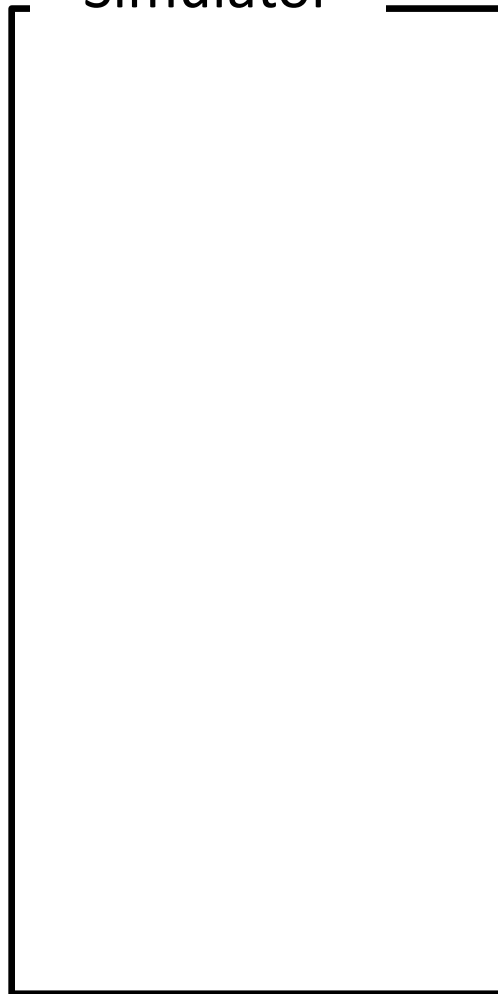
$$E \leftarrow \prod_{i=1}^n E_i = \prod_{i=1}^n H_1(t)^{x_i} \cdot H_2(t)^{H_3(t, m_i) x_i}$$
$$\Sigma \leftarrow (E, t)$$

## ④ Overview of Our Security Proof

Challenger  
(MCL)



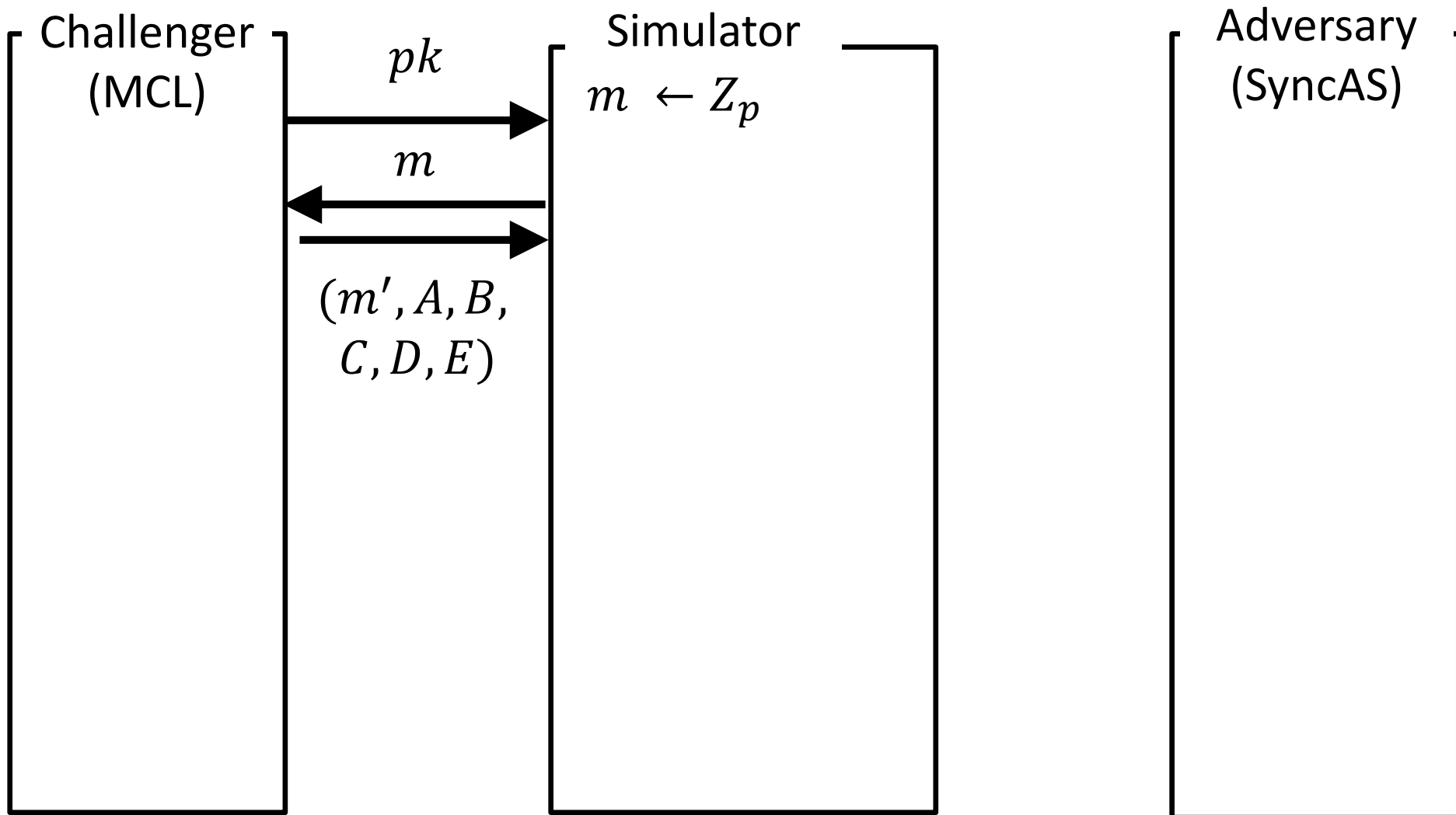
Simulator



Adversary  
(SyncAS)

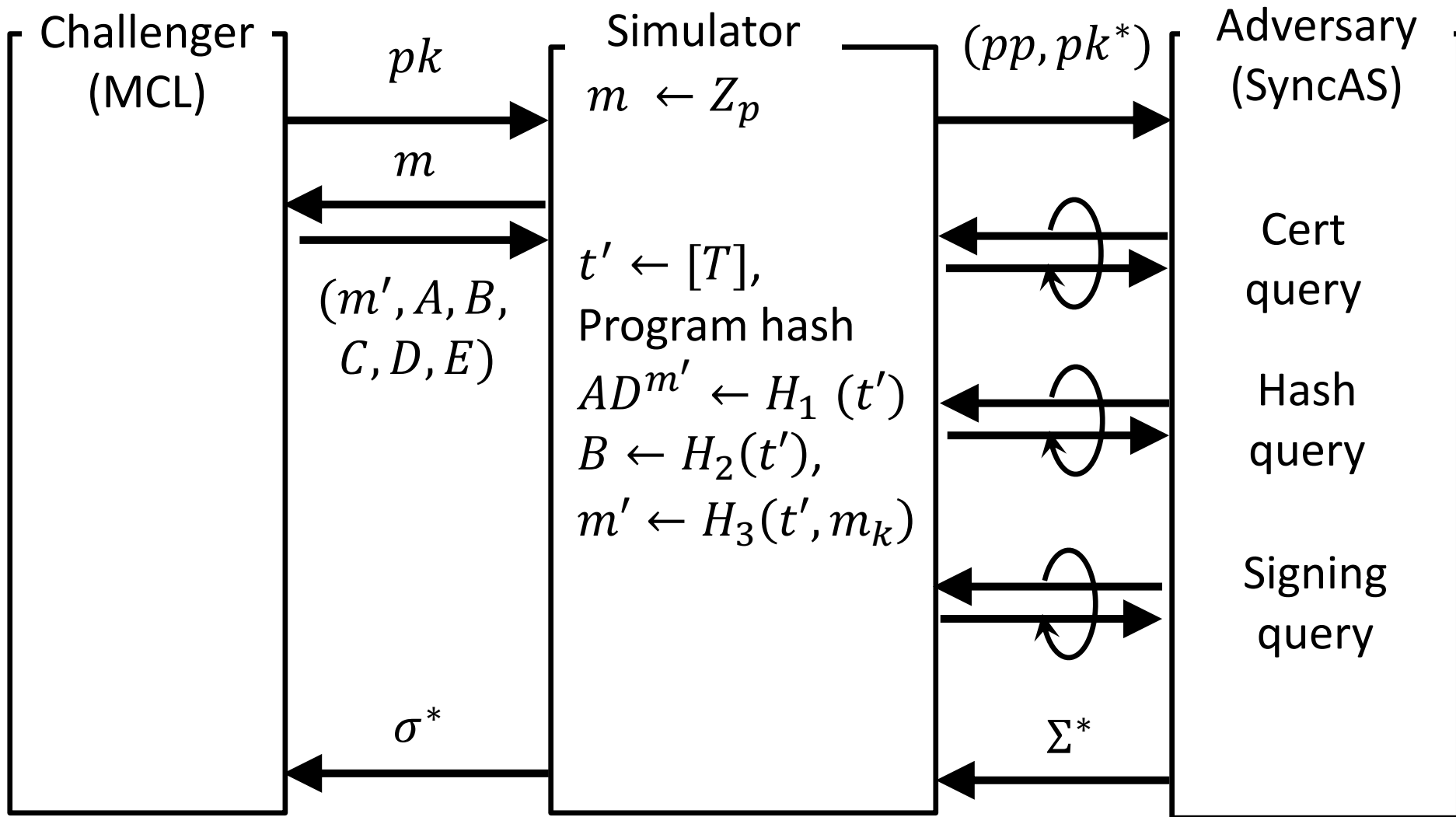


## ④ Overview of Our Security Proof





## ④ Overview of Our Security Proof



## ④ Conversion from MCL Sig to LLY SyncAS

MCL signature

$$\sigma_i \leftarrow (m_i', A_i, B_i \leftarrow A_i^{y_i}, C_i \leftarrow A_i^{y_i}, D_i \leftarrow C_i^{y_i}, E_i \leftarrow A_i^{x_i} D_i^{m_i' x_i} B_i^{m_i x_i})$$



I. Force signers to use same  $m_i', A_i, B_i, C_i, D_i$ .

$$\Sigma \leftarrow \left( m', A, B, C, D, E = \prod_{i=1}^n E_i = \prod_{i=1}^n \left( (AD^{m'})^{x_i} B^{m_i x_i} \right), t \right)$$



II. Change  $AD^{m'}$  to  $H_1(t)$ ,  $B$  to  $H_2(t)$ ,

$m_i$  to  $H_3(t, m_i)$ .

$$\Sigma \leftarrow \left( E = \prod_{i=1}^n E_i = \prod_{i=1}^n H_1(t)^{x_i} H_2(t)^{H_3(t, m_i) x_i}, t \right)$$

# References 1/2

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[AGH10] Ahn, Green, and Hohenberger.

Synchronized aggregate signatures: new definitions, constructions and applications. (ACM CCS 2010)

[BGLS03] Boneh, Gentry, Lynn, and Shacham.

Aggregate and verifiably encrypted signatures from bilinear maps. (EUROCRYPT 2003)

[CL04] Camenisch and Lysyanskaya.

Signature schemes and anonymous credentials from bilinear maps. (CRYPTO 2004)

[GR06] Gentry and Ramzan.

Identity-based aggregate signatures. (PKC 2006)

# References 2/2

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- [LMRS04] Lysyanskaya, Micali, Reyzin, and Shacham.  
Sequential aggregate signatures from trapdoor permutations.  
(EUROCRYPT 2004)
  
- [LLY 13] Lee, Lee, and Yung.  
Aggregating CL-signatures revisited: Extended functionality and better efficiency. (FC 2013)
  
- [LRSW99] Lysyanskaya, Rivest, Sahai, and Wolf.  
Pseudonym systems. (SAC1999)
  
- [PS16] Pointcheval and Sanders.  
Short Randomizable Signatures. (CT-RSA 2016)
  
- [PS18] Pointcheval and Sanders.  
Reassessing security of randomizable signatures. (CT-RSA 2018)

# Appendix: Modified CL Signature

$$pp = (p, G, G_T, e)$$

KeyGen( $pp$ )

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$$e(C, Y) = e(D, g) ?$$

$$e(AB^m D^{m'}, X) = e(E, g) ?$$

Sign( $sk, m$ )

$$m' \leftarrow Z_p, A \leftarrow G^*, B \leftarrow A^y$$

$$C \leftarrow A^z, D \leftarrow C^y, E \leftarrow A^x B^{mx} D^{m'x}$$

$$\sigma \leftarrow (m', A, B, C, D, E)$$